Basic question 1: Complexity

Here is a simple sorting algorithm that works on linked lists:

```
function sort(xs : list of integers) → list of integers:
    queue = new maximum priority queue (using a binary heap)
    for each x in xs:
        add x to queue

    result = new linked list of integers
    while queue is not empty:
        remove the maximum element y from queue
        insert y at the front of result

    return result
```

What is the asymptotic complexity of this function in the number of elements N of `xs`?

Write your answer in O-notation. Be as exact and simple as possible. Justify why the complexity of the function has this order of growth.

Answer

Complexity: _____

Justification (you can also add notes directly in the code above):

Basic question 2: Sorting

Perform a quicksort partitioning of the following array, using the element at position 3 as pivot:

0	1	2	3	4	5	6	7	8
15	74	63	51	32	48	89	91	27

Note: quicksorting the left and right parts of the partition is not part of this question.

Answer

Write down how the array looks after the partitioning:

0	1	2	3	4	5	6	7	8

What sequence of swaps did you make when partitioning the array?

If you used a different algorithm from that of your course, explain it here:

Basic question 3: Basic data structures

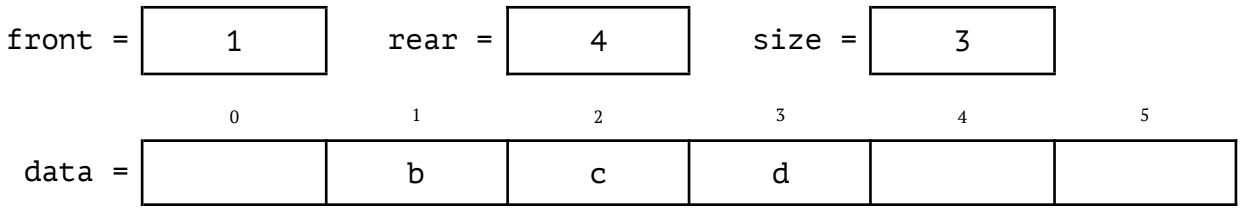
We have a class Queue that is implemented as a circular array. Internally, it has an array `data` and these integer variables:

- `front`: index of the front of the queue,
- `rear`: index of the rear of the queue,
- `size`: number of elements in the queue.

The class has two public methods, `enqueue` and `dequeue`:

```
class Queue:
    method enqueue(item : int)
    method dequeue() → int
```

If we create a queue q with a capacity of six, add the four elements a, b, c, and d, and then remove one, the internal representation looks like this:

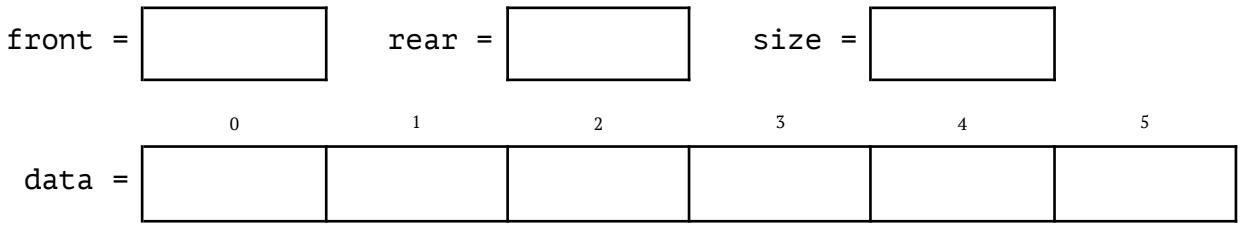


Now we execute the following sequence of method calls:

```
q.dequeue()
q.enqueue(e)
q.enqueue(f)
q.dequeue()
q.enqueue(g)
```

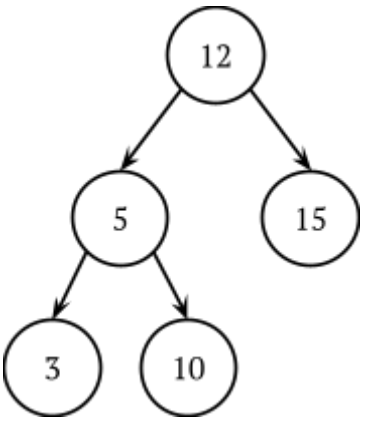
How does the internal state of q look after these operations?

Answer



Basic question 4: Search trees

Insert 8 into the following AVL tree using the AVL insertion algorithm.



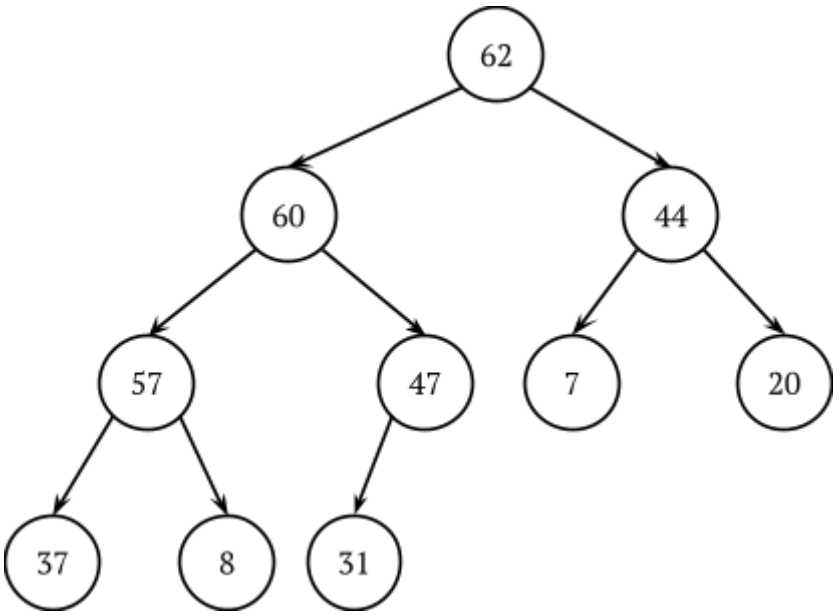
Answer

How does the tree look after you have inserted 8, but before rebalancing?

How does the tree look after rebalancing?

Basic question 5: Priority queues

You are given a maximum priority queue implemented as the following binary heap:



Answer

How does the above heap look when represented as an array?
You can choose the 0-based or 1-based version.

0	1	2	3	4	5	6	7	8	9	10

Now remove the maximum element from the priority queue. How does the array look afterwards?

0	1	2	3	4	5	6	7	8	9	10

Basic question 6: Hash tables

Suppose we have the following *open-addressing* hash table using *linear probing*.
Our hash function is the identity function $h(x) = x \pmod{10}$.

0	1	2	3	4	5	6	7	8	9
8	1		23	14	4	5		18	9

The hash table was created by adding the elements 1, 4, 5, 8, 9, 14, 18, 23 **in an unknown order**.
The array has never been resized, and no elements have been deleted.
In what orders could the elements have been added?

A) 9, 8, 1, 14, 23, 4, 5, 18

B) 18, 23, 14, 1, 8, 4, 5, 9

C) 1, 14, 23, 18, 4, 9, 5, 8

D) 18, 14, 4, 1, 9, 8, 5, 23

E) 18, 9, 1, 23, 4, 5, 8, 14

Determine which of these orders are possible (there may be several, or even none).
For the others, explain why they are impossible.

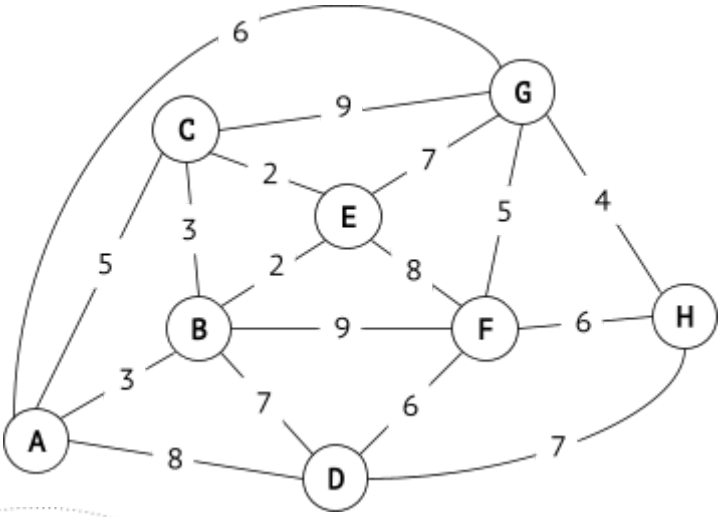
Answer

Which orders are possible? _____

Explain why the others are impossible:

Basic question 7: Graphs

You are given the graph to the right.

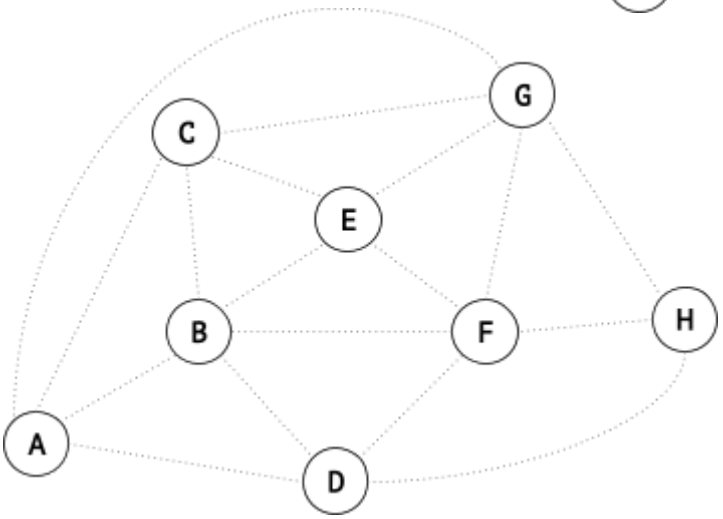


Draw the minimum spanning tree (MST).

Choose either Prim's or Kruskal's algorithm.
In which order does the algorithm
add the edges to the MST?

Answer

Draw the MST by filling
the dotted edges:



Chosen algorithm (Prim or Kruskal): _____

Starting vertex (only if needed): _____

MST edges added in order (not all cells have to be used):

1		4		7	
2		5		8	
3		6		9	

Basic question 8: Abstract data types

The following code implements a common abstract data type (ADT) using a common data structure.

```
class MysteriousDataStructure:
    a : Node

    method f() -> int:
        x = this.a.value
        this.a = this.a.next
        return x

    method g(y : int):
        n = new Node
        n.value = y
        n.next = this.a
        this.a = n

class Node:
    value : int
    next : Node
```

Answer

Which ADT is implemented here? _____

Give the methods `f` and `g` better names:

`f` = _____

`g` = _____

Advanced question 1: Complexity

The following is a very meaningless function F that takes an integer *n* as input:

```
function F(n):
    S = new set (using an AVL tree)
    for i from 0 to n:
        for j from 0 to n:
             S.add(i + j)

    for each a in S:
        for each b in S:
            doSomething(a, b)
```

Asymptotically, what is the **space complexity** and the **time complexity** of F in the input *n*? Assume that `doSomething` is O(1) in both time and space.

Write your answers in O-notation. Be as exact and simple as possible. Explain the key reasons why each complexity has the order of growth you give.

Part 1

Space complexity: _____

Time complexity: _____

Justification:

Part 2

Suppose we replace `i + j` by `i*n + j` in the line where we add to S. What is your answer now?

Space complexity: _____

Time complexity: _____

Justification:

Advanced question 2: Hash tables

Here is an *incorrect* algorithm for deleting an item in a hash table using *linear probing/open addressing*:

- Find the position of the item to be deleted. Suppose it is at index *i* in the array.
- If the item is the *last* item in the cluster, just remove it (i.e., put *null/None* there).
- Otherwise, find the last item in the cluster and move it to index *i*.

For example, suppose we have the following hash table, using the hash function *h(x) = x (mod 10)*:

0	1	2	3	4	5	6	7	8	9
28	38		3		5	15		8	18

If we delete 18, it will move 38 from position 1 (the end of the cluster) to position 8, producing:

0	1	2	3	4	5	6	7	8	9
28			3		5	15		38	18

Or if we delete 3, the 3 will be replaced by a *null/None*.

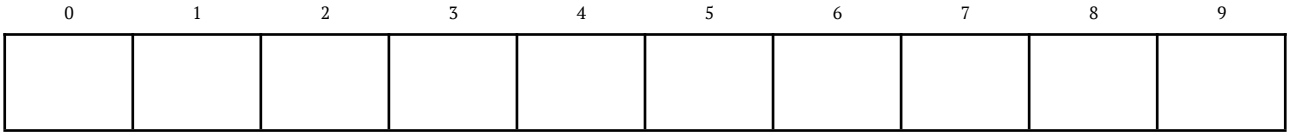
This algorithm is incorrect. After deleting an item, the hash table may no longer be valid. Your task is to find an example where the algorithm goes wrong. That means:

- You start with a valid hash table
- You delete an item using the algorithm above
- Afterwards, an item is stored in an incorrect position

Once you find an example, you can ***write your answer on the next page***.

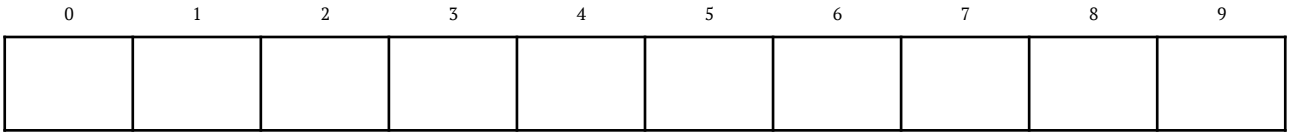
Advanced question 2, answer

Which (valid) hash table do you start from?
Use the hash function $h(x) = x \pmod{10}$.



Which item do you delete? _____

How does the hash table look afterwards?



Which item is now in the wrong place? _____

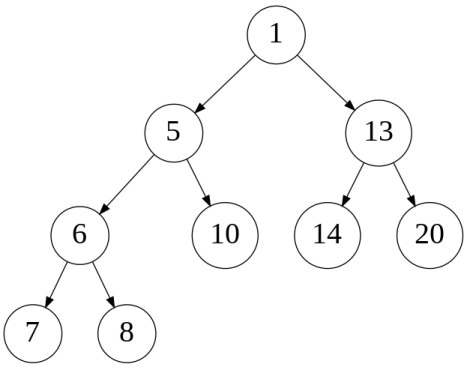
Advanced question 3: Strange trees

A *strange tree* is a data structure storing a set of keys. It is a binary tree, but not a binary search tree.

A strange tree is either *null/None* or has a key and left and right children satisfying the following:

- The key is *less than* the keys in its subtrees (i.e., a strict version of the heap property holds).
- All the keys in the left subtree are *less than* all the keys in the right subtree.

Here is an example of a strange tree (pointers to *null/None* nodes are omitted):



Your task is to write a function:

```
function contains(node : StrangeTree, key : int) → bool
```

which *searches* for a key in a strange tree. It should return *true* if the key is found or *false* otherwise.

You may assume that the `StrangeTree` class looks like this:

```
class StrangeTree:
    key   : int          // the key stored in this node
    left  : StrangeTree  // the left child
    right : StrangeTree  // the right child
```

Or, if you prefer Haskell:

```
data StrangeTree = Empty | Node Int StrangeTree StrangeTree
```

Note:

- Your solution should take *O(H)* time in terms of the height *H* of the tree.
- You may answer using pseudocode or Java/Python/Haskell.
- You can define helper functions if you want.

Write your answer on a separate sheet of paper.

Advanced question 4: Data structure design

Two words are *anagrams* if you can rearrange the letters of one to get the other. Examples of anagrams are (1) *cat* and *act*; (2) *dare*, *dear* and *read*; (3) *wolves* and *vowels*; and (4) *algorithm* and *logarithm*.

Your task is to design a data structure for finding all anagrams of a given word. The data structure represents a collection of words. It should have two operations:

- `add(word : string)`: add a new word to the data structure
- `anagrams(word : string) → list[string]`: given a word, return a list of all words that are anagrams of it. The word itself should not be included in the list, only its anagrams.

Here is an example of using the data structure:

```
words = new anagram data structure
// add some words to the data structure
words.add("cat"); words.add("act")
words.add("dare"); words.add("dear"); words.add("read")
// now find some anagrams
words.anagrams("cat")    // should return the list ["act"]
words.anagrams("dear")   // should return the list ["dare", "read"]
words.anagrams("monkey") // should return an empty list []
```

The operations should have the following complexities (or better):

- add should take O(log(*n*)) time where *n* is the size of the data structure
- anagrams should take O(log(*n*) + *r*) time where *r* is the number of anagrams returned

You can assume that the following function already exists – you don't need to implement it:

- `sortLetters(word : string) → string`: sort the letters of a string alphabetically. For example, `sortLetters("dear")` and `sortLetters("dare")` both return "`ader`".

Assume that all string operations (equality, comparison, `sortLetters`) take **constant time** O(1).

You may use existing data structures and algorithms in your solution – you don't need to implement those yourself. You may answer using pseudocode or Java/Python/Haskell.

Write your answer on a separate sheet of paper.