Basic question 1: Quicksort pivot selection

Consider the following 9-element array:

0	1	2	3	4	5	6	7	8
23	47	17	82	58	94	32	70	66

You want to partition the whole array, as the first step in running Quicksort.

a) For each of the following pivot selection strategies, specify what element will be the pivot, and how big the left and right parts will be after performing the partitioning.

Strategy	**Pivot index**	**Pivot value**	**Left part size**	**Right part size**
First-index	0			
Middle-index	4			
Last-index	8			

b) Which of the strategies gives the best ~~partitioning~~ partition ***for this specific array***?

c) Give an example of a 5-element array where that strategy (i.e., your answer to question (b)) will give the ***worst possible ~~partitioning~~ partition***:

0	1	2	3	4

d) What will now be the pivot value in (c)?

Basic question 2: Hash tables

To the right is an open addressing hash table containing binary numbers. It uses modular compression (using the modulo operator) and linear probing (with probing constant 1, as usual).

The hash function is the ***number of ones*** in the binary number.

(*Note*: the numbers in parentheses are *not* the hash values – they are just the decimal representations of the binary numbers, so that, e.g., $11100111_2 = 231_{10}$.)

0	11111110 (254)
1	00000000 (0)
2	00010100 (20)
3	11111111 (255)
4	00000010 (2)
5	
6	11100111 (231)
7	10110111 (183)

a) Give one possible ordering in which the numbers could have been added to the hash table. It's enough to use the decimal representations in your answer.

   (***Note***: no elements have been deleted from the table.)

The table is almost overfull, so we have to resize it.

b) Insert all the numbers into the table of size 11 to the right, ***in the same order*** as the order you gave in (a) above.

   You don't have to write both the binary and decimal representation – either of them is fine.

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Basic question 3: Unbalanced BSTs

You create a binary search tree by inserting the first N positive integers in *reverse order* into an initially empty tree using the standard insertion procedure.

So, you start by inserting N into an empty tree, then you insert N–1, …, 3, 2, and finally 1.

a) What is the root node value? _____

b) What are the value(s) of all the leaf nodes (if any)? _____
(***Note***: a leaf node is a node with only null children.)

c) What is the time complexity of building the final tree? _____

Now you want to delete all elements, one at the time, using the standard deletion procedure.

d) In which order should you delete the elements to get the ***worst possible*** time complexity?

_____

e) What is the time complexity of deleting all elements in this order? _____

f) In which order should you delete the elements to get the ***best possible*** time complexity?

_____

g) What is the time complexity now? _____

Note: In questions (c), (e) and (g) we mean the *asymptotic time complexity* of the whole procedure (i.e., adding/deleting all elements, not just one single element), in terms of the number of elements N. Please answer in O-notation, and as tight and simple as possible.

Basic question 4: Priority queues

Here is a broken binary max-heap with 10 elements represented by a dynamic array:

0	1	2	3	4	5	6	7	8	9	10
23	14	20	16	8	13	7	2	7	5	null

a) Two array elements are misplaced and need to swap to make this a correct heap. What are their indices? Specify their values as well for clarity.

Indices: _____ Values: _____

Now, answer the following two questions on the *corrected* heap (after you have swapped the misplaced elements):

b) Which cells of the array would be modified (written to) in the worst case when adding a new element to the corrected heap? (Specify all indices.)

_____

c) Which cells of the array would be modified (written to) when deleting the maximum from the corrected heap using the standard algorithm? (Specify all indices.)

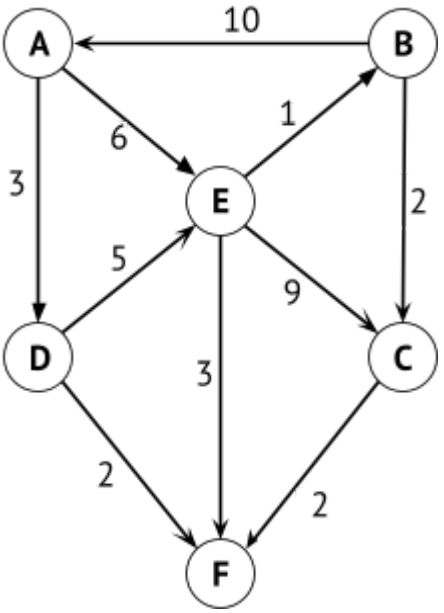
_____

Note: This is performed on the original corrected array, i.e., (b) never happened.
Hint: Assigning null to a cell counts as a modification.

Basic question 5: Graphs

We run this familiar piece of code on the graph to the right:

```
visited = new set of nodes
agenda = new min-priority queue of pairs of cost and node
agenda.add( (0, A) )
while agenda is not empty:
    (cost, node) = agenda.removeMin()
    if not visited.contains(node):
        visited.add(node)
        for e in outgoingEdges(node):
            agenda.add( (cost + e.weight, e.target) )
```



What are the contents of the set *visited* and the priority queue *agenda*, initially and after each of the first three iterations of the while loop? (Some values are pre-filled for explanatory purposes.)

Iteration	***visited***	***agenda (in order of priority)***
before the loop	—	(0, A)
after iteration 1	A	(3, D), (6, E)
after iteration 2		
after iteration 3		
after iteration 4		

Basic question 6: Complexity

Below are two (mostly nonsense) functions, each taking an integer array as a parameter.

State the asymptotic time complexity of each function **in terms of the length *N* of the array**.

In each case, **briefly state** how you concluded this (you may do this by annotating the programs).

***Notes*:**

- Assume that all arithmetic operations and array indexing take O(1) time.
- Answer in O-notation, and as tight and simple as possible.

Algorithm 1

Complexity: _____

Justification:

```
function fun1(arr):
    i = 0  [Mistake in question. Should be i = 1.]
    while i < length(arr):
        for j = 0 to length(arr):
            arr[i] = arr[i] + arr[j]
        i = i × 2
```

Algorithm 2

Complexity: _____

Justification:

```
function fun2(arr):
    result = 0
    i = length(arr) − 1
    while i ⩾ 0:
        result = result × arr[i]
        if arr[i] % 2 == 1:
            i = i − 1
        else:
            i = i − 2
    return result
```

Advanced question 7: Word chains

You are given a list of words and two letters of the alphabet (called *start* and *goal*), and your task is to come up with an algorithm that finds a word chain between *start* and *goal*, using as *few* words as possible.

A *word chain* is a sequence of words from the given list, where the first letter of each word is the last letter of the previous word in the sequence. In addition, the first word must begin with the starting letter and the last word must end with the goal letter.

Example: For the word list [***ape***, ***axis***, ***data***, ***dog***, ***end***, ***salad***, ***sinus***], and ***start=a***, ***goal=g***, there are two word chains with three words in them. One is [***ape***, ***end***, ***dog***] and the other is [***axis***, ***salad***, ***dog***], and your algorithm should return either of those. There are many other possible chains for this list of words, but they all have more than three words.

So, the following call:

```
wordchain(["ape","axis","data","dog","end","salad","sinus"], "a", "g")
```

should return either `["ape","end","dog"]` or `["axis","salad","dog"]`.

Also state the asymptotic worst case time complexity of your algorithm, in terms of the number of words *N*. Please answer in O-notation, and as tight and simple as possible. Motivate your answer.

You can use every data structure and algorithm in the course, but be clear about how they are used.

Advanced question 8: Mystery data structure

Here is a chunk of pseudocode defining two classes:

```
class Y:
    a: integer
    b: Y

class X:
    t: array of Y
    k: integer

    def f(x):
        i = x % length(t)
        q = t[i]

        if q is null:
            return

        if q.a == x:
            t[i] = q.b
            k = k − 1
            return

        while q.b is not null:
            if q.b.a == x:
                q.b = q.b.b
                k = k − 1
                return
            q = q.b
```

What does this strange code represent?

a) What common data structure is defined by the class **X**?
b) What do the instance variables **t** and **k** represent?
c) What does the method **f** do?

Be as precise as possible and use correct technical terms taught in the course.

Advanced question 9: Anagram finder

An anagram is a word or phrase formed by rearranging the letters of another word or phrase, e.g., **"listen"** is an anagram of **"silent"**. Your assignment is to make a data structure for finding anagrams in large sets of words, with two operations:

- **addWord(w)**: adds the word **w** to the data structure.
- **anagrams(w)**: returns a list of all anagrams of **w** that have been added to the data structure (this should include **w** itself only if it has been added).

Example: Assume that I run **addWord("silent"); addWord("listen"); addWord("stilton")** on an empty instance of the data structure. Then **anagrams("listen")** and **anagrams("enlist")** should both return **["silent", "listen"]** (or **["listen", "silent"]**). In contrast, **anagrams("tonsil")** should return the singleton list **["stilton"]**. [Mistake in question. "Stilton" is not an anagram of "tonsil".]

a) Describe how you implement the data structure and the two operations. Make it as efficient as you can. You can use every data structure and algorithm in the course, but be clear about how are they are used.

   Hint: the words **"cabs"** and **"scab"** are both anagrams of the character sequence **"abcs"**.

b) What is the complexity of the **anagrams** operation in terms of the number N of separate anagram lists in the data structure (in the example, there would be two anagram lists, one for **"listen"/"silent"** and one for **"stilton"**). Assume that the number of characters K in the word searched for does not exceed $\log(N)$, meaning that K is in $O(\log(N))$.

   Explain your reasoning.

   Note: remember that comparing two strings is linear in the length of the strings, i.e., $O(K)$ if K is the number of characters in the strings.