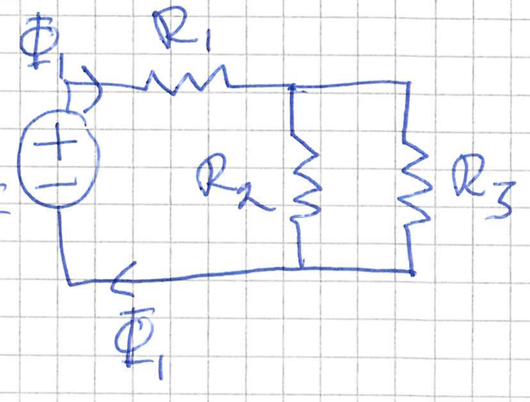
EEN155 tentamen 20/8 2025
kortfattade lösningar

① a) Beräkningsschema:



$$\begin{cases} A_1 = A_2 = A_3 \equiv A \\ \mu_1 = \mu_2 = \mu_3 = \mu_0 \cdot \mu_r \\ l_1 = l_3 = 3 l_2 \end{cases}$$

R_2 & R_3 är parallellkopplade.

R_1 i serie med $R_2 // R_3$

$$\Rightarrow \quad R_1 = R_3 = 3 R_2$$

$$R_{tot} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = R_2 \left(3 + \frac{3}{4}\right)$$

$$R_2 = \frac{l_2}{\mu_0 \mu_r \cdot A} = \frac{5 \cdot 10^{-2}}{4\pi 10^{-7} \cdot 2 \cdot 10^{3} \cdot 3 \cdot 10^{-4}} \approx 6{,}63146 \ 1/H$$

$$\Rightarrow \quad R_{tot} \approx 2{,}486796 \cdot 10^{5} \ 1/H$$

$$\begin{cases} NI = R_{tot} \cdot \Phi_1 \\ \Phi_1 = B_1 A \end{cases} \Rightarrow \quad I = \frac{R_{tot} \cdot B_1 A}{N} \approx \underline{\underline{0{,}23 \text{ ampere}}}$$

b) Gauss lag för magnetfält

2) a) $\mathbb{T} = \mathbb{m} \times \mathbb{B}$, $\mathbb{m} = NIA \cdot \mathbb{n}$

$\Rightarrow \mathbb{T} = NIA\ \mathbb{n} \times \mathbb{B}$ med $\mathbb{n} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$

$\mathbb{B} = B_x\,\hat{x} + B_y\,\hat{y}$

$\left.\begin{array}{l} \hat{x} \times \hat{x} = \hat{y} \times \hat{y} = 0 \\ \hat{x} \times \hat{y} = \hat{z}\ ,\ \hat{y} \times \hat{x} = -\hat{z} \end{array}\right\} \Rightarrow \mathbb{n} \times \mathbb{B} = (-\sin\varphi\, B_y - \cos\varphi\, B_x)\,\hat{z}$

$\therefore\ \mathbb{T} = NIA \cdot (\sin\varphi \cdot B_y + \cos\varphi \cdot B_x) \cdot (-\hat{z})$

$\varphi = 30^\circ \Rightarrow \sin 30^\circ = 0{,}5\ \ \&\ \cos 30^\circ = \frac{\sqrt{3}}{2} \simeq 0{,}866$

$\Rightarrow \mathbb{T} = 10 \cdot 1{,}5 \cdot 0{,}12^2 \cdot (0{,}5 \cdot 0{,}4 + 0{,}866\ 0{,}1) \cdot (-\hat{z})$

$\simeq -0{,}0619\,\hat{z}$ Nm

$\therefore$ Vridmomentet är $\simeq \underline{0{,}062\ \text{Nm}}$, riktat $\underline{\otimes}$

b) $\mathbb{B}$ beror här ej av rumsposition $\Rightarrow \Phi = \mathbb{B} \cdot \mathbb{A}$

genom slingen

$\left.\begin{array}{l} \mathbb{B} = B_x \cos\omega t\,\hat{x} + B_y \sin\omega t\,\hat{y} \\ \mathbb{A} = A \cdot \hat{y} \end{array}\right\} \Rightarrow \boxed{\Phi = B_y \cdot A \cdot \sin\omega t}$

$\hat{x} \cdot \hat{y} = 0$

$\hat{y} \cdot \hat{y} = 1$

endast $\mathbb{B}$:s $\hat{y}$-komponent bidrar

$u_{ind} = -N \frac{d\Phi}{dt} = -N B_y A \omega \cos\omega t$

$t = 2{,}5 \cdot 10^{-5}\,s \Rightarrow |u_{ind}| = 5 \cdot 2 \cdot 10^{-3} \cdot \pi\, 0{,}04^2 \cdot 10^5 \cdot \cos(10^5 \cdot 2{,}5 \cdot 10^{-5}) \simeq \underline{4{,}0\,V}$

3

Elektriska flödet genom en yta:

$$\Phi_e = \int_{yta} \mathbb{D} \cdot d\mathbb{A} = \varepsilon_0 \int_{yta} \mathbb{E} \cdot \mathbb{n}\, dA$$

där $\mathbb{n}$ = normalvektor till ytan, riktas ut från ytan om ytan del av slutet område.

Uppgiften: $\mathbb{E}$ riktat i $+\hat{x}$ ⇒ passerar genom kubens sidor i yz-planet.

Vid $x=0$ är $\mathbb{n} = -\hat{x}$, vid $x = 0{,}01\,m$ är $\mathbb{n} = +\hat{x}$.

$\mathbb{E}$ oberoende av y & z ⇒ integration över en kubsida ger $\mathbb{E} \cdot \mathbb{n} \cdot$ arean av sidan.

⇒ Totala flödet genom kuben blir:

$$\Phi_{e,tot} = \Phi_e(x=0) + \Phi(x=0{,}01) =$$

$$= \{25000 \cdot (1+2\cdot 0) \cdot \underbrace{(-1)}_{\hat{x}\cdot(-\hat{x})} + 25000 \cdot (1+2\cdot 0{,}01) \cdot \underbrace{(+1)}_{\hat{x}\cdot\hat{x}}\} \cdot$$

$$\cdot\, \varepsilon_0 \cdot \underbrace{0{,}01^2}_{\text{arean hos en kubsida}}$$

$\approx 4{,}43 \cdot 10^{-13}$ coulomb

Gauss lag för el. fält: $Q_{inne} = \Phi_{e,tot} \approx \underline{\underline{4{,}4 \cdot 10^{-13}\,C}}$

(positiv laddning)

4a) $V_f = 440\,V$ $V_T = 350V$ $T_L = B \cdot \omega_r = 0.6\,\omega_r$

i) varvtal?

$V_T = R_a i_a + \lambda \omega_r$

$$\left\{ \begin{array}{l} T_e = \lambda i_a = T_L = B\omega_r \\ \Rightarrow i_a = \dfrac{B\omega_r}{\lambda} \end{array} \right.$$

λ?
$V_f = 440$, $R_f = 200$
$i_f = \dfrac{440}{200} = 2.2A$
$\Rightarrow \lambda = 1.63\,Wb$

$V_T = R_a \dfrac{B}{\lambda}\omega_r + \lambda\omega_r$

$$V_T = \omega_r\left(\frac{R_a B}{\lambda} + \lambda\right) \Rightarrow \omega_r = \frac{V_T}{\frac{R_a B}{\lambda} + \lambda} = \frac{350}{\frac{0.22 \cdot 0.6}{1.63} + 1.63} = 204.561 \text{ rad/s}$$

$$n_r = \omega_r \cdot \frac{30}{\pi} = 1953.4 \text{ rpm}$$

ii) i_a?

$$i_a = \frac{B \cdot \omega_r}{\lambda} = \frac{0.6 \cdot 204.561}{1.63} = 75.3\,A$$

ok < märkström 127A.

iii) motemk?

$$e_a = \lambda\omega_r = 1.63 \cdot 204.561 = 333.4\,V$$

iv) se ovan

$$P_{cu,f} = R_f i_f^2 = 200 \cdot 2.2^2 = 968\,W$$

v)

$$P_{mek} = \omega_r \cdot T_e = \omega_r \cdot \lambda \cdot i_a = 204.561 \cdot 1.63 \cdot 75.3 = 25107.61\ldots W$$

$$\approx 25.1\,kW$$

4b)

V_T = märkspänning = 440V

i_a = märkström = 127 A

$T_L = B \cdot \omega_r = 0.6 \cdot \omega_r$ Nm

$v_f, i_a, \lambda = ?$

(1) $V_T = R_a i_a + \lambda \omega_r$

$T_e = \lambda i_a = T_L = B \cdot \omega_r \Rightarrow \omega_r = \frac{i_a \lambda}{B}$ (2)

(2) i (1)

$V_T = R_a i_a + \lambda \frac{i_a \lambda}{B}$

$\lambda^2 \frac{i_a}{B} = V_T - R_a i_a \Rightarrow \lambda = \pm \sqrt{\frac{V_T - R_a i_a}{i_a / B}} = \sqrt{\frac{440 - 0.22 \cdot 127}{127/0.6}} = 1.395\ldots$ Wb

λ i (2) $\Rightarrow \omega_r = \frac{i_a \cdot \lambda}{B} = \frac{127 \cdot 1.395}{0.6} = 295.275$ rad/s

$\underline{n_r} = \omega_r \cdot \frac{30}{\pi} = \underline{2819.7 \text{ rpm}}$

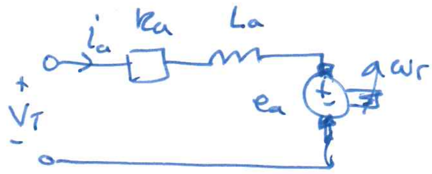
$\lambda = k_\lambda \cdot i_f$ vid 2.2A är $\lambda = 1.63$

$\Rightarrow k_\lambda = \frac{\lambda}{i_f} = \frac{1.63}{2.2} = 0.74091$ Wb/A

$\underline{i_f} = \frac{\lambda}{k_\lambda} = \frac{1.395}{0.74091} = \underline{1.883 \text{ A}}$

4c)

ankarkretsens ekvivalenta kretsschema

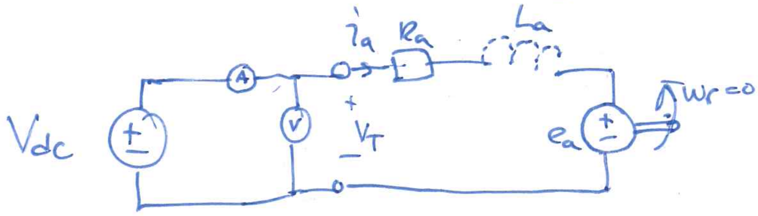


$$V_T = R_a i_a + L_a \frac{di_a}{dt} + \underbrace{a\omega r}_{e_a}$$

För att mäta Ra bör förutsättningarna vara sådana att varken La eller e_a inverkar. Det gör de ej om

- V_T = dc-spänning i stationärtillstånd $\Rightarrow \frac{di_a}{dt} = 0$
- $e_a = 0$, det sker om $V_f = 0 \Rightarrow i_f = 0 \rightarrow a = 0$
  genom att inte magnetisera maskinen. Låt fältspänningen vara noll, vilket ger noll fältström och noll länkat flöde.
  Då utvecklas inget vridmoment $(T_e = a \cdot i_a = 0 \cdot i_a = 0)$
  och rotorn kommer inte att accelerera.
  Varvtalet förblir noll och $(e_a = a \cdot \omega r = 0 \cdot 0 = 0.)$

Så läss på en dc-spänning som ger en ankarström nära märkström



- mät ankarström (A) och ankarspänning (V)

$$R_a = \frac{V_T}{I_a}$$

För att mäta La behöver även här $e_a = 0$ så $i_f = 0$, $V_f = 0$.

Samma uppkoppling som ovan men byt ut dc-spänningskällan mot en AC-spänningskälla med känd fix frekvens f_s: Vac

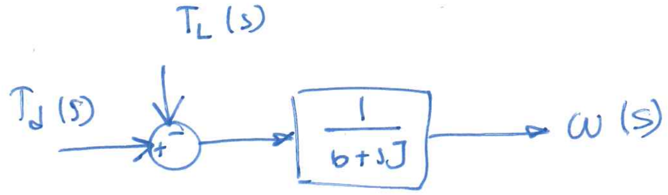
Mät $V_{T,rms}$, $i_{a,rms}$

$$|Z| = \frac{V_{T,rms}}{i_{a,rms}} = \sqrt{R_a^2 + \omega^2 L_a^2}$$

där $\omega = 2 \cdot \pi \cdot f_s$, lös ut

$$L_a = \sqrt{\frac{\left(\frac{V_{T,rms}}{i_{a,rms}}\right)^2 - R_a^2}{(2\pi f_s)^2}}$$

5a)



nedan härledning behöver ej redovisas

$$J\frac{d\omega(t)}{dt} = T_d(t) - b\omega(t) - T_L(t)$$

$$\Big\downarrow \mathcal{L}$$

$$Js\,\omega(s) = T_d(s) - b\,\omega(s) - T_L(s)$$

$$T_d(s) - T_L(s) = Js\,\omega(s) + b\,\omega(s) = \omega(s)\left(Js + b\right)$$

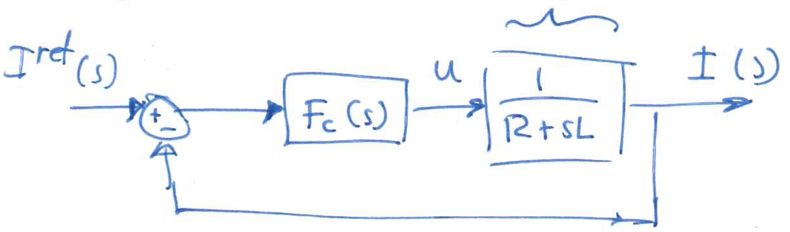
$$\omega(s) = \left[T_d(s) - T_L(s)\right]\cdot\left(\frac{1}{b+sJ}\right)$$

5b) R, L i serie EEN155 2025-08-20

i) • $u = Ri + L\frac{di}{dt} \xrightarrow{\mathcal{L}} U(s) = RI(s) + LsI(s) = I(s)\cdot(R+sL)$

- ström regulator $F_c(s)$ $I(s) = U(s)\cdot\underbrace{\left(\frac{1}{R+sL}\right)}$

• blockdiagram av systemet



process $G(s) = \frac{1}{R+sL}$

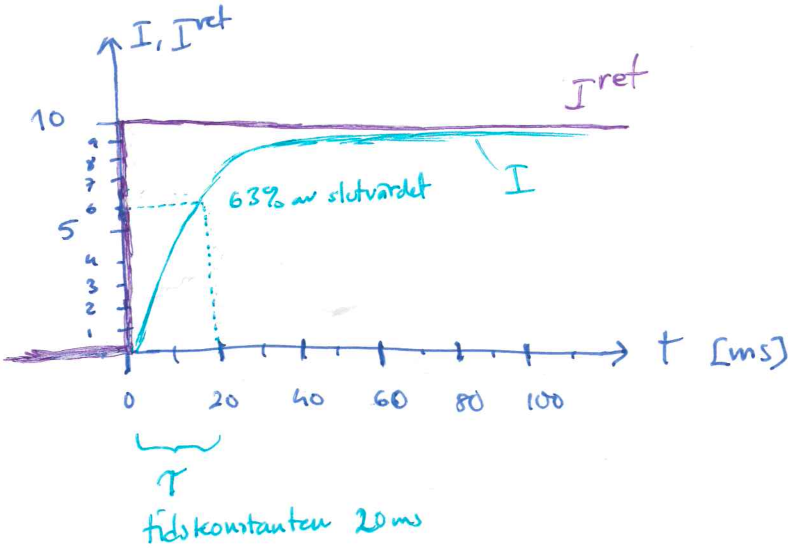
- överföringsfunktion för det slutna systemet

$$H(s) = \frac{I(s)}{I^{ref}(s)} = \frac{F_c(s)\cdot G(s)}{1 + F_c(s)\cdot G(s)} = \left\{\begin{matrix}\text{1:a ordn system}\\ \Rightarrow \text{ingen över-svång}\end{matrix}\right\} = \frac{\alpha}{s+\alpha} = \frac{\alpha/s}{1+\alpha/s}$$

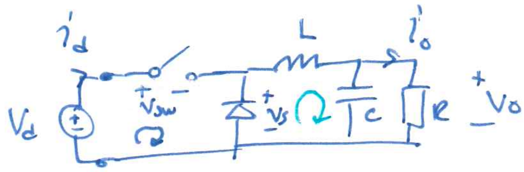
$$\Rightarrow F_c(s)\cdot G(s) = \frac{\alpha}{s} \Rightarrow F_c(s) = \hat{G}^{-1}(s)\cdot\frac{\alpha}{s} = (\hat{R}+s\hat{L})\cdot\frac{\alpha}{s}$$

$$\Rightarrow F_c(s) = \underbrace{\hat{L}\alpha}_{K_p} + \underbrace{\frac{\hat{R}\alpha}{s}}_{K_i}$$

ii) $\tau = 20\,ms$ $I^{ref} = 10\,A$



6a) $D = f(I_d, I_o)$?



$V_{L\,ave} = 0$

$$V_{L\,ave} = \frac{1}{T}\int_0^T V_L(t)\,dt = 0$$

$$= \frac{1}{T}\int_0^{DT} V_L(t)\,dt + \frac{1}{T}\int_{DT}^{T} V_L(t)\,dt =$$

$$= \frac{1}{T}\int_0^{DT} V_d - V_o\,dt + \frac{1}{T}\int_{DT}^{T} -V_o\,dt =$$

$$= \frac{1}{T}(V_d - V_o)(DT) + \frac{1}{T}(-V_o)(T - DT) =$$

$$= \frac{1}{T}(V_d DT - V_o DT) + \frac{1}{T}(-V_o T + V_o DT) =$$

$$= \frac{1}{\cancel{T}}(V_d D\cancel{T} - V_o \cancel{DT} - V_o \cancel{T} + \cancel{V_o DT}) = V_d D - V_o = 0 \Rightarrow V_o = D V_d$$

$$D = \frac{V_o}{V_d} = \frac{8}{24} = \frac{1}{3}$$

KVL

$-V_d + V_{sw} + V_s = 0$

sw on: $V_{sw} = 0$, $V_d = V_s$

sw off: $V_s = 0$, $V_d = V_{sw}$

KVL

$-V_s + V_L + V_o = 0$

sw on: $V_s = V_d$, $V_L = V_d - V_o$

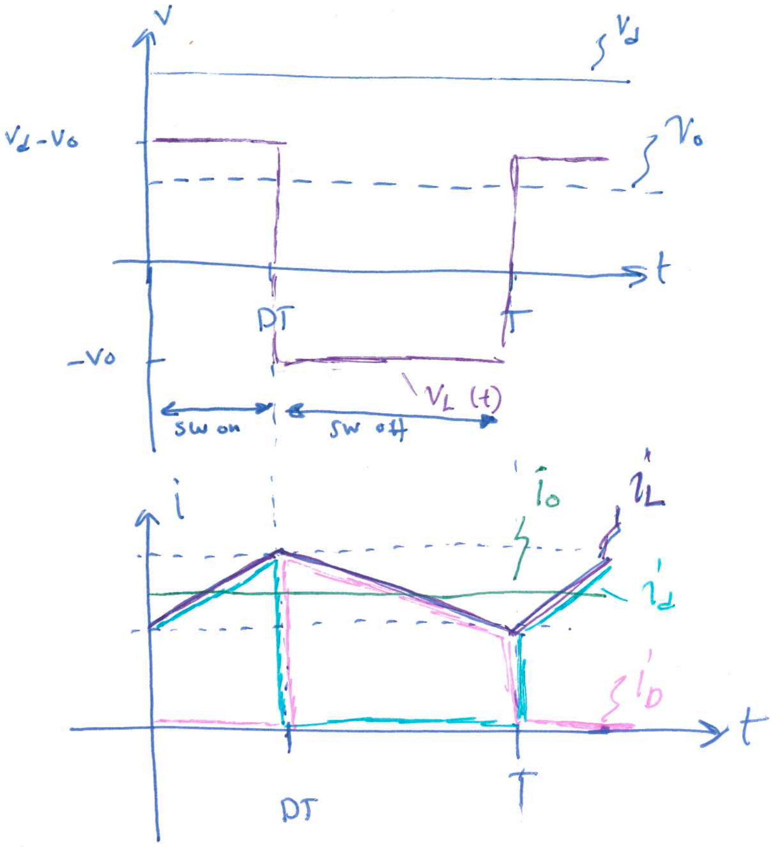
sw off: $V_s = 0$, $V_L = -V_o$

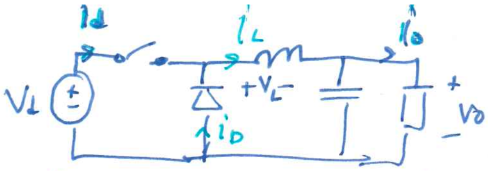
- antag • stationärtillstånd, ideala komponenter
  • stor C, CCM,

$P_{in} = P_{ut}$

$$\cancel{V_d} I_d = V_o I_o = D \cancel{V_d} I_o \Rightarrow I_d = D I_o \Rightarrow \underline{\underline{D = \frac{I_d}{I_o}}}$$

6b) v_L, i_o, i_d, i_L, i_D





i_d = in-ström

i_o - utström

$v_L = L \frac{di_L}{dt}$

$v_L > 0 = \frac{di_L}{dt} > 0$

$v_L < 0 \Rightarrow \frac{di_L}{dt} < 0$

6c) CCM om $\frac{\Delta i_L}{2} < I_{Lave} = I_o$ $\qquad I_o = \frac{V_o}{R} = \frac{8}{2} = 4\,A$

Δi_L ? $\quad v_L = L\frac{di_L}{dt} = L\frac{\Delta i_L}{\Delta t} \Rightarrow \Delta i_L = \frac{v_L \cdot \Delta t}{L} = \left\{\begin{matrix} sw \\ on \end{matrix}\right\} = \frac{(V_d - V_o)(DT)}{L} =$

$\Delta i_L = \frac{(V_d - V_o)\cdot D}{f \cdot L} = \frac{(24-8)\cdot\frac{8}{24}}{40\cdot10^3\cdot100\cdot10^{-6}} = 1{,}33\ldots\ A$

$\frac{\Delta i_L}{2} = \frac{2}{3} = 0.666\ldots\ A < 4A = I_o \Rightarrow$ Ja, CCM

7a)

$V_n = 440$ V rms 50 Hz

$T_L = 16$ Nm $n_r = 1210$ rpm $i_{ph} = 3.7A$ $\cos\varphi = 82\%$

i) $\underline{P_{mek}} = \omega_r \cdot T_e = n_r \cdot \frac{\pi}{30} \cdot T_L = 1210 \cdot \frac{\pi}{30} \cdot 16 = 2027,374459$ W

$\approx \underline{2027,4 \text{ W}}$

ii) $P_{förl} = P_{in} - P_{ut} = P_{el} - P_{mek}$

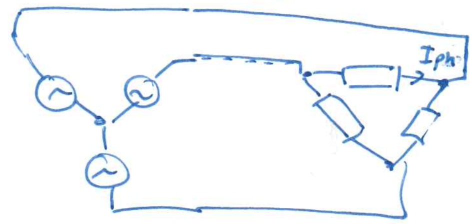
$P_{el} = 3 \cdot V_{ph} \cdot I_{ph} \cdot \cos\varphi = 3 \cdot \frac{440}{\sqrt{3}} \cdot 3,7 \cdot 0.82 = 2312.2185$ W

$\underline{P_{förl}} = 2312.218546 - 2027.374459 = \underline{289.84 \text{ W}}$

iii) $\underline{\eta} = \frac{P_{ut}}{P_{in}} = \frac{P_{mek}}{P_{el}} = \frac{2027.374459}{2312.218546} = 0.876809...$

$\approx \underline{87,7\%}$

7b) $P=4$ $U_n = 440$ rms 60 Hz Δ-kopplad

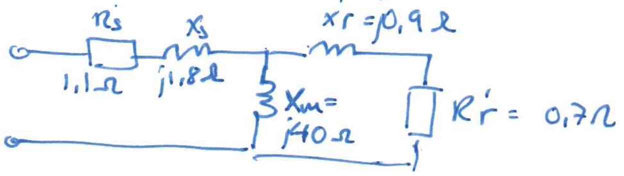


$R_s = 1{,}1\,\Omega$ $R'_r = 0.7\,\Omega$ $X_m = 40\,\Omega$

$X_s = 1.8\,\Omega$ $X'_r = 0.9\,\Omega$

i) i start ögonblicket $n_r = 0 \Rightarrow s = \frac{n_s - n_r}{n_s} = 1$

$\frac{R'_r}{s} = R'_r$



ii) $I_L = \sqrt{3} \cdot I_{ph}$ i delta-kopplad last

$$Z_{eq} = R_s + jX_s + \frac{jX_m(R'_r + jX'_r)}{jX_m + R'_r + jX'_r} = 1{,}1 + j1.8 + \frac{j40(0{,}7 + j0{,}9)}{j40 + 0{,}7 + j0{,}9} =$$

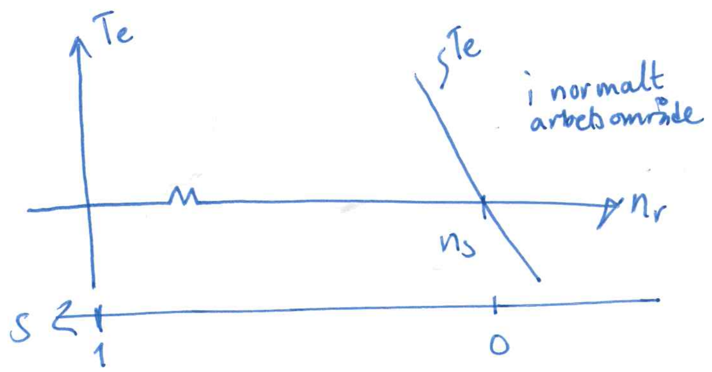
$$= 1{,}769736043 + j2{,}691651228 = 3{,}22110\ldots\angle 56.68139\ldots^\circ$$

$$I_{ph} = \frac{U_h}{Z_{eq}} = \frac{440\angle 0^\circ}{3.22110\ldots\angle 56.68^\circ} = 136{,}59929\ldots\angle -56.68^\circ \text{ A}$$

$$|I_L| = \sqrt{3} \cdot |I_{ph}| = \sqrt{3} \cdot 136.59929 = \underline{236{,}5969251\ldots \text{ A}}$$

$$f_{rotor} = s \cdot f_{stator} = 1 \cdot 60 = \underline{60 \text{ Hz}}$$

7c)



8a) $\hat{i}_{sd}, \hat{i}_{sq}$?

$\beta = 132°$

$I_{mag} = ?$

$\hat{i}_{sd} = I_{mag} \cdot \cos\beta$

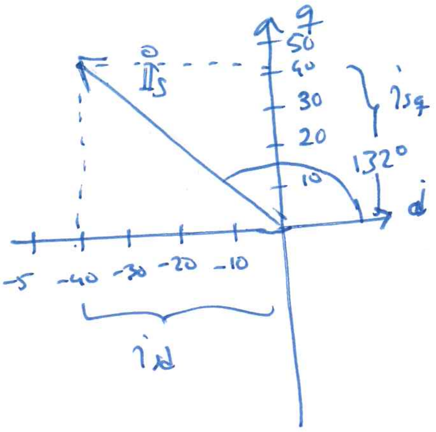
$\hat{i}_{sq} = I_{mag} \sin\beta$

$P_{cu,fort} = 3 R_s \cdot I_{rms}^2 = 5.1 \text{ kW}$

$\Rightarrow I_{rms} = \sqrt{\dfrac{P_{cu,ford}}{3R_s}} = \sqrt{\dfrac{5.1 \cdot 10^3}{3 \cdot 0.88}} = 43.95 \text{ A}$

$I_{mag} = \sqrt{2} \cdot I_{rms} = \sqrt{2} \cdot 43.95 = 62{,}16 \text{ A}$

$$\boxed{\begin{aligned} \hat{i}_{sd} &= 62.16 \cdot \cos 132° = -41.59 \text{ A} \\ \hat{i}_{sq} &= 62.16 \cdot \sin 132° = 46.19 \text{ A} \end{aligned}}$$



8b) $f_s = ?$

$n_r = 3400 \text{ rpm} \quad n_p = \frac{4}{2} = 2$

$\omega_s = 2\pi f_s \Rightarrow \boxed{f_s = \dfrac{\omega_s}{2\pi}} = \{\omega_s = \omega_r\} = \dfrac{\omega_r}{2\pi} = \{\omega_r = \Omega_r n_p\} = \dfrac{\Omega_r n_p}{2\pi}$

$f_s = \dfrac{\Omega_r n_p}{2\pi} = \left\{\Omega_r = n_r \cdot \dfrac{\pi}{30}\right\} = n_r \cdot \dfrac{\pi}{30} \cdot \dfrac{n_p}{2\pi} = \dfrac{n_r \cdot n_p}{60} = \dfrac{n_r \cdot 2}{60} = \dfrac{3400}{60} \cdot 2 =$

$\underline{\underline{f_s = 113{,}33.. \text{ Hz}}}$