CHALMERS

Examination in MCC015: Superconducting Devices – Fundamentals and Applications

M-building

Wednesday June 1st 2016 14.00 – 18.00

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi – 031 772 3318,

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

You have to answer to all problems

Total credits: 15.0: 7 credits passed, 10 credits well passed, 13 credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam. You will get, from home assignments and lab reports, max 3 credits if exam score is < 4 and max 2 credits if exam score is > 4.

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1. SHORT PROBLEMS (3 credits total):

1.1. Draw the I-V characteristic of the junction for Q<<1 (strong damping). By considering the mechanic analog of the junction draw the time dependent voltage across the junction for $I \cong I_C$ and $I >> I_C$, with I_C being the critical current of the junction. (0.5 credits)

1.2 Consider a short Josephson junction of length $L << \lambda_J$ in the z direction and an external magnetic field applied in the plane of the junction in the x direction. Draw the spatial variation of the Josephson current as a function of z if the applied external flux is $2\Phi_0$ and $(5/2)\Phi_0$, where Φ_0 is the flux quanta. (0.5 credits)

1.3 Consider a dc SQUID with negligible inductance and with $Ic_1 \neq Ic_2$. Sketch the maximum supercurrent through the SQUID as a function of the normalized external applied magnetic flux Φ_{ext}/Φ_0, where $\Phi_0=h/2e$. Which are the maximum and minimum values of the total critical current across the SQUID? (0.5 credits)

1.4 Consider a junction with R= 5 Ω, C=1 pF and Ic= 1 μA. Calculate the quality factor for this junction and draw the I-V characteristic. Imagine to place an external capacitor C_e in parallel to a Josephson junction with these parameters. Which is the minimum value of C_e for the I-V characteristic of the junction to become hysteretic? (0.5 credits)

1.5 On what length scale does microwave radiation penetrate a bulk superconductor and how does it depend on frequency? (0.5 credits)

1.6 What is the π - Josephson junction? How it can be observed experimentally? (0.5 credits)

2. Josephson junction in a magnetic field

Consider a square Josephson junction in (y,z) plane and an external magnetic field H applied along the y direction. Thickness of the junction is $d = t + \lambda_1 + \lambda_2$, where t is insulating barrier thickness, λ_1, λ_2 – London penetration depth of both superconductors.

a) Derive the expression for the gradient of phase difference across the barrier:

$$\frac{\partial\varphi}{\partial z} = \frac{2\pi\mu_0 d}{\Phi_0} \cdot H_y(z)$$

What is the characteristic penetration depth of magnetic field in the junction? (assume both magnetic field and current through the junction are small) (1.5 credits).

b) Assume junction size, L is much smaller than the Josephson penetration depth. Derive the dependence of the maximum Josephson current as a function of magnetic flux across the junction area (1.5 credits).

3. Two-fluid model of a superconductor

Consider the two fluids model for a superconductor. Derive the expression of the complex conductivity of a superconductor in presence of an external ac electromagnetic field and discuss the equivalent circuit. (3 points)

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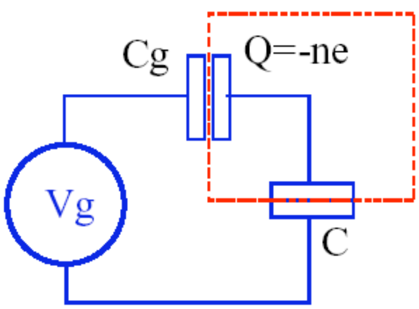
4. Single electron transistor

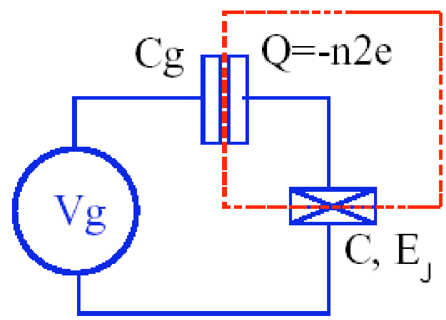
Figure below shows the schematic of a single electron box SEB (a) and of a single Cooper–pair box SCB (b).

a) For both physical systems, draw and comment the diagram of the energy of the box as a function of the gate voltage Vg, and for different numbers, n, of the charges on the box (omit the terms not dependent on n). Discuss the main differences between the two energy diagrams. (1.5 credits)

b) For both physical systems draw and comment the dependence of the charge on the box as a function of the gate voltage (i.e. Coulomb staircase). Discuss what determines the smearing of the staircase for a SEB and a SCB. (1.5 credits)



(a)



(b)

5. Dc SQUID

1. Describe operation of the dc SQUID as a fluxmeter. Derive expression for the voltage in the SQUID using RSCJ model and show that the voltage is a periodic function of the external magnetic flux when bias current is set close to the critical current of the SQUID. (1 credit)

2. Discuss three main sources of noise in the dc SQUID. Derive the expression for the voltage noise as a function of SQUID inductance and normal resistance. (1 credit)

3. Describe the operation of the dc SQUID in the flux-locked loop (FLL) electronics and explain how it can linearize the voltage-to-flux response of the SQUID. Derive the expression for the FLL magnetic flux gain. (1 credit)

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