

**Examination in MCC015: Superconducting Devices – Fundamentals and Applications****M-building****Wednesday May 31<sup>st</sup> 2017 14.00 – 18.00**

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi – 031 772 3318,

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

**You have to answer to all problems**

Total credits: 15.0: 7 credits passed, 10 credits well passed, 13 credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam. You will get, from home assignments and lab reports, max 3 credits if exam score is  $< 4$  and max 2 credits if exam score is  $> 4$ .

**1. SHORT PROBLEMS (3 credits total):**

1.1 Draw the current voltage characteristic of a  $S_1$ -I- $S_2$  Josephson junction and describe the different tunneling currents. (0.5 credit)

1.2 Draw the dependence of the critical current  $I_C$ , and Josephson energy,  $E_J$  of the Josephson junction as a function of phase,  $\varphi$  in the range  $0-2\pi$ . Using mechanical analog, discuss the physical meaning of two states at  $\varphi=0, 2\pi$  and  $\varphi=\pi$  (0.5 credit)

1.3 Consider the two fluids model for a superconductor. Describe the equivalent circuit for a superconductor in presence of microwave radiation. (0.5 credit)

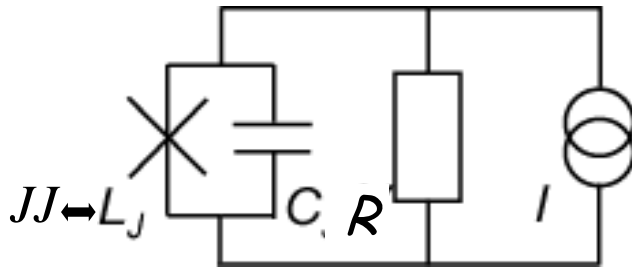
1.4 How does the I-V curve change for an SET when the electrodes become superconducting? (0.5 credit)

1.5 Draw a dependence of the maximum critical current modulation depth of a dc SQUID as a function of screening parameter  $\beta_L$  (assume that  $I_{C1} = I_{C2}$ ). (0.5 credit)

1.6 Describe how S-I-N junction can be used as a thermometer. How the thermal responsivity at a fixed current bias can be defined? (0.5 credit)

**2. RCSJ model of the Josephson junction**

1. The equivalent circuit for a Josephson junction is represented in Figure 1 (RCSJ model).



- a) Discuss the physical origin of the capacitance and resistance in the model. Write the equation of the circuit in terms of the superconducting phase difference across the junction. (1 credit)
- b) Consider Josephson junction with  $R_N = 500 \Omega$ ,  $C = 1\text{pF}$  and  $I_C = 1 \mu\text{A}$ . Imagine to place an external resistor  $R_e$  in parallel to a Josephson junction with these parameters. Which is the value of  $R_e$  for the I-V characteristic junction to become non hysteretic? (1 credit)
- c) Draw the time dependent voltage across the junction for  $I \cong I_C$  and  $I \gg I_C$ , with  $I_C$  being the critical current of the junction. Consider junction with  $Q \ll 1$ . (1 credit)

**3. Wave propagation in the Josephson junction**

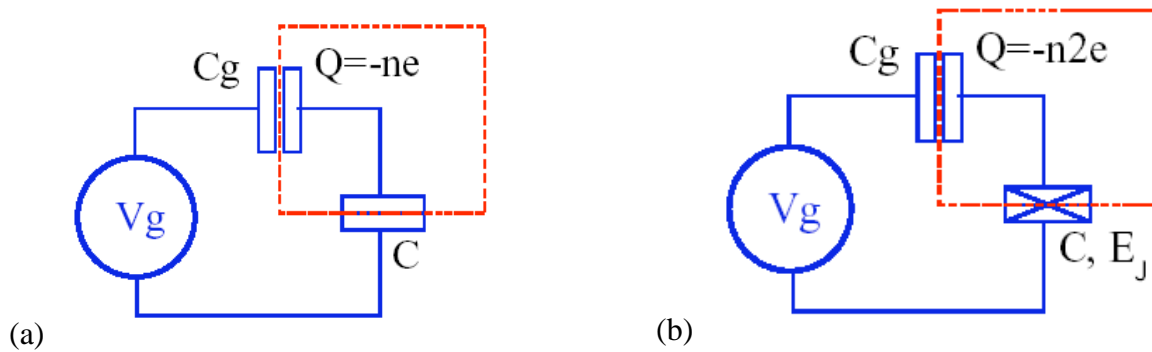
- a) Derive the Sine – Gordon equation for a Josephson junction in terms of the phase difference. Explain the physical reason why the value of the phase velocity in the junction is much lower than the light velocity (for usual junction’s parameters) (1.5 credits)
- b) Specify the Sine-Gordon equation for the case of short junction and explain the role of plasma frequency. What happens in case of long junction? Write a dispersion relationship for this case. (1.5 credits)

4. Single electron transistor

Figure below shows the schematic of a single electron box SEB (a) and of a single Cooper-pair box SCB (b).

a) For both physical systems, draw and comment the diagram of the energy of the box as a function of the gate voltage  $V_g$ , and for different numbers,  $n$ , of the charges on the box (omit the terms not dependent on  $n$ ). Discuss the main differences between the two energy diagrams. (1.5 credits)

b) For both physical systems draw and comment the dependence of the charge on the box as a function of the gate voltage (i.e. Coulomb staircase). Discuss what determines the smearing of the staircase for a SEB and a SCB. (1.5 credits)



## 5. SQUIDS

1. Consider a single Josephson junction that is connected by a superconducting loop with inductance  $L$ . Show that in the absence of external magnetic field the current in the loop can take on values given by

$$I = -I_c \sin\left(\frac{2\pi LI}{\Phi_0}\right)$$

(1 credit)

2. Draw schematically the dependence of total magnetic flux in the superconducting loop with single Josephson junction as a function of external magnetic flux for different values of screening parameter  $\beta_L = 0, 1$  and  $10$ . (1 credit)
3. Describe the operation of the dc SQUID in the flux-locked loop (FLL) electronics and explain how it can linearize the voltage-to-flux response of the SQUID. Derive the expression for the FLL magnetic flux gain. (1 credit)



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