Examination in MCC015: Superconducting Devices – Fundamentals and Applications

Wednesday June 5th 2019 14.00 – 18.00

SB Multisal, Sven Hultins gata 8

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi 031 772 3318

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

You have to answer all problems

Total credits: **15**: **7** credits passed, **10** credits well passed, **13** credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam. You will get, from home assignments and lab reports, max **3** credits if exam score is **< 4** and max **2** credits if exam score is **> 4**.

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1. SHORT PROBLEMS (3 credits):

1.1 Draw the dependence of the critical current I_C, and Josephson energy, E_J of a Josephson junction as a function of phase, φ in the range of 0-2π. Using mechanical analog, discuss the physical meaning of two states at φ=0,2π and φ=π. **(0.5 credits)**

1.2 Draw the magnetic field dependence of the Josephson current for a rectangular short junction (which has a uniform current distribution at zero external field ($\Phi(B_{ext})$)=0). Derive the current distribution when the applied magnetic flux is $\Phi(B_{ext})= 2\,\Phi_0$, where Φ_0 is the flux quanta. **(0.5 credit)**

1.3 On what length scale does microwave radiation penetrate a bulk superconductor and how does it depend on frequency? **(0.5 credit)**

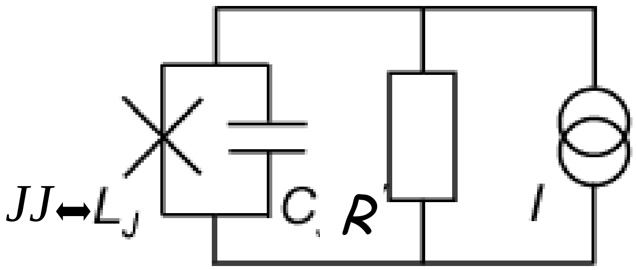
1.4 Define the characteristic length which determines a short and a long Josephson junction behavior. **(0.5 credit)**

1.5 A dc SQUID with a voltage modulation depth of ΔV_{pp}=30 μV is included into the flux-locked loop with negative feedback. The feedback loop resistance is R_f= 1 kΩ and mutual inductance is M_f= 1 nH. Calculate external magnetic flux that can be applied to produce $1/8\Phi_0$ in the SQUID loop. Voltage gain of the preamplifier is G_{LNA}=10000. **(0.5 credit)**

1.6 Explain the differences between a Cooper-pair box and a Transmon qubit. What makes one of them better than the other? **(0.5 credit)**

2. RCSJ model of the Josephson junction (3 credits)

The equivalent circuit for a Josephson junction is represented in Figure 1 (RCSJ model).



a) Discuss the physical origin of the capacitance and resistance in the model. Write the equation of the circuit in terms of the superconducting phase difference across the junction. **(1 credit)**

b) Consider a Josephson junction with R_N = 500 Ω, C = 1pF and I_C = 1 µA. Imagine to place an external resistor R_e in parallel to a Josephson junction with these parameters. Which is the value of R_e for the I-V characteristic junction to become non-hysteretic? **(1 credit)**

c) Draw the time dependent voltage across the junction for $I \cong I_C$ and $I >> I_C$, with I_C being the critical current of the junction. Consider junction with Q << 1. **(1 credit)**

3. Two-fluid model of a superconductor (3 credits)

Consider the two fluids model for a superconductor. Derive the expression of the complex conductivity of a superconductor in presence of an external ac electromagnetic field and discuss the equivalent circuit.

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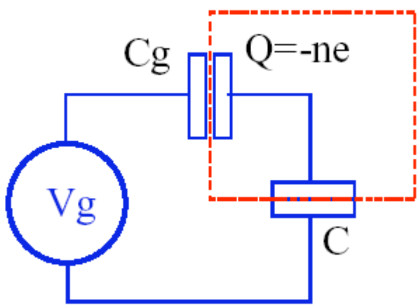
4. Single electron transistor (3 credits)

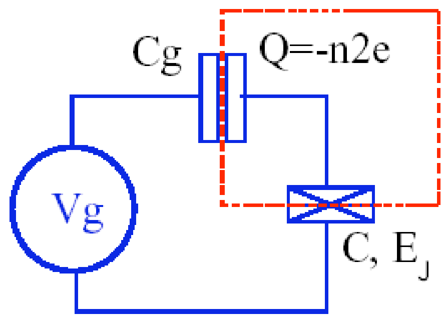
Figure below shows the schematic of a single electron box SEB (a) and of a single Cooper–pair box SCB (b).

a) For both physical systems, draw and comment the diagram of the energy of the box as a function of the gate voltage Vg, and for different numbers, n, of the charges on the box (omit the terms not dependent on n). Discuss the main differences between the two energy diagrams. **(1.5 credits)**

b) For both physical systems draw and comment the dependence of the charge on the box as a function of the gate voltage (i.e. Coulomb staircase). Discuss what determines the smearing of the staircase for a SEB and a SCB. **(1.5 credits)**



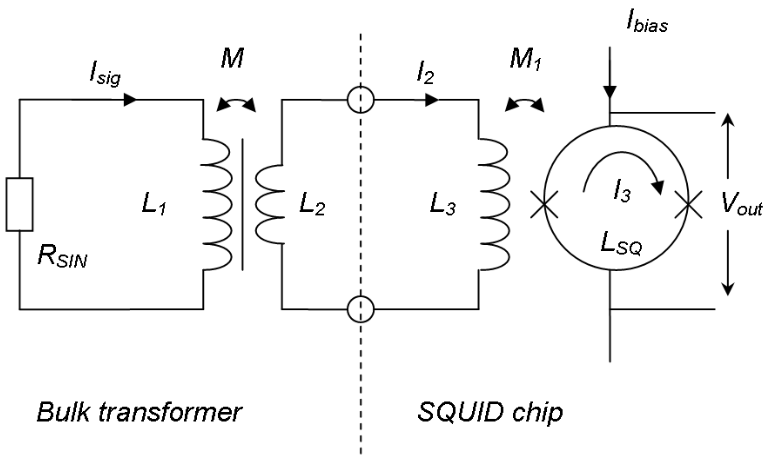
(a)



(b)

5. SQUID as a current amplifier (3 credits)

Dc SQUID can be used to measure changes in various physical quantities that can be converted into magnetic flux in the SQUID loop. For electrical current measurements, a double superconducting transformer scheme is typically used as shown in the figure below. Signal current, *Isig* is flowing in the first transformer (L_1, M, L_2) that is inductively connected to a superconducting flux transformer (L_3, M_1, L_{SQ}) that is in turn inductively coupled to the SQUID loop:



Derive maximum current sensitivity in A/Φ_0 of such double-transformer. Neglect resistive losses in the input transformer (R_{SIN} = 0) and assume that the coupling efficiencies α and α_i ~1. How high should be the inductance of the input transformer L_1 to obtain current noise level below 100 fA/Hz$^{1/2}$ (1 fA = 10^{-15} A) assuming SQUID inductance $L_{SQ} = 10^{-10}$ H and SQUID flux noise $S_\Phi^{1/2} = 10^{-6}\ \Phi_0/\mathrm{Hz}^{1/2}$? Discuss how this inductance can be practically realized.

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