

Examination in MCC015: Superconducting Devices – Fundamentals and Applications

Wednesday June 3rd 2020, 14.00 – 18.00

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi 031 772 3318

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

You have to answer all problems

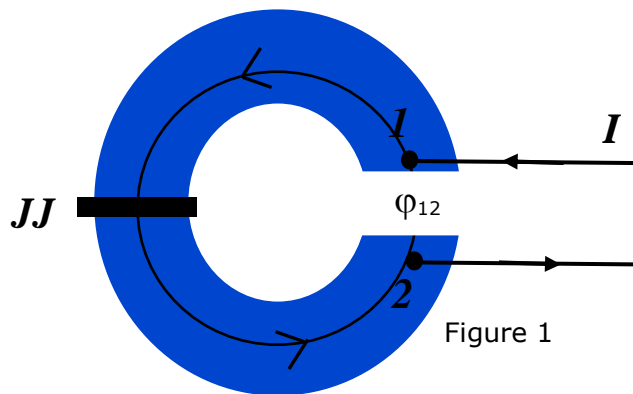
Total credits: **15**: 7 credits passed, **10** credits well passed, **13** credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam.

You will get, from home assignments and lab reports, max **3** credits if exam score is **< 4** and max **2** credits if exam score is **> 4**.

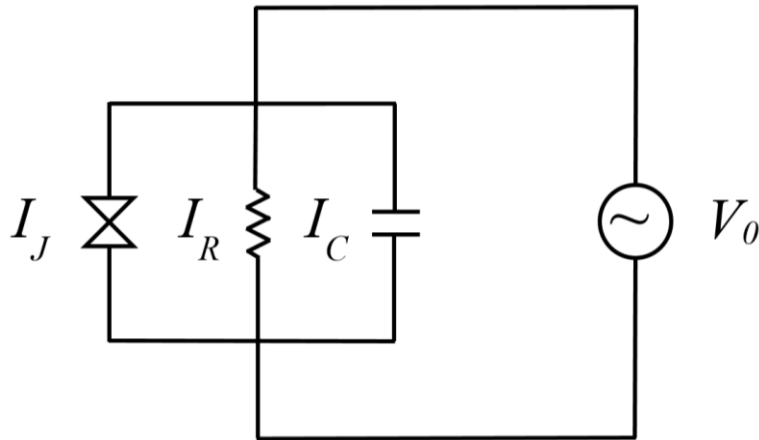
1. Josephson junction in the ring (3 credits):

Consider a Josephson junction with a critical current $I_c = 1 \mu\text{A}$ incorporated in a superconducting ring of inductance $L = 1 \text{ nH}$ (see Figure 1). On the side opposite to the junction, a small piece is cut out, as illustrated in the figure. An external current source, connected across the opening, supplied a current $I = 0.4 \mu\text{A}$. Find the phase difference of the macroscopic wave function between the points 1 and 2, φ_{12} .



2. RSCJ model (3 credits)

- a) Consider a constant voltage source V_0 connected across the Josephson junction. Using the RCSJ model, find the total current through the junction $I_{TOT}=I_C+I_J+I_R$ averaged over one time period Φ_0/V_0 . Plot the I-V characteristic of such voltage biased junction. Does it depend on the quality factor of the Josephson junction? Motivate your answer. **(2 credits)**



- b) Draw the time dependent voltage across the junction for $I \cong I_C$ and $I \gg I_C$, with I_C being the critical current of the junction. Consider junction with $Q \ll 1$. **(1 credit)**

3. dc SQUID (3 credits)

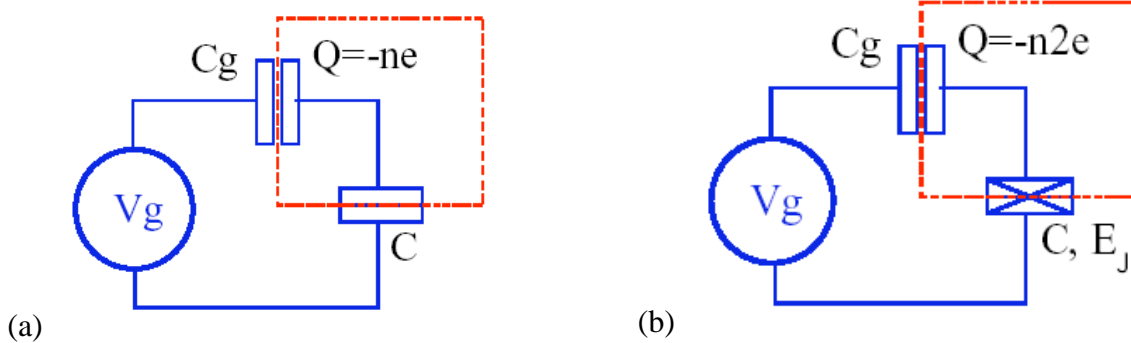
Derive magnetic flux dependence of the maximum critical current of the asymmetric DC SQUID with $I_{C,1} > I_{C,2}$ assuming negligible screening ($\beta_L \ll 1$). Show that the ratio of the maximum to minimum total critical current for the two-junction SQUID with negligible self-induced flux is given by $(\alpha + 1) / (\alpha - 1)$, where $\alpha = I_{C,1} / I_{C,2}$.

4. Single electron transistor (3 credits)

Figure below shows the schematic of a single electron box SEB (a) and of a single Cooper-pair box SCB (b).

a) For both physical systems, draw and comment the diagram of the energy of the box as a function of the gate voltage V_g , and for different numbers, n , of the charges on the box (omit the terms not dependent on n). Discuss the main differences between the two energy diagrams. (1.5 credits)

b) For both physical systems draw and comment the dependence of the charge on the box as a function of the gate voltage (i.e. Coulomb staircase). Discuss what determines the smearing of the staircase for a SEB and a SCB. (1.5 credits)



5. Kinetic Inductance (3 credits)

Consider a superconducting strip of width w and thickness $t \ll \lambda$, where λ is London penetration depth. Derive the equation for the kinetic inductance L_K per unit length resulting from the motion of the paired electrons by comparing the energy stored in an inductor having a current I flowing through it with the kinetic energy related to the motion of the paired electrons.