Examination in MCC015: Superconducting Devices – Fundamentals and Applications

Wednesday June 2nd 2021, 14.00 – 18.00

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi 031 772 3318

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

You have to answer all problems

Total credits: **15**: **7** credits passed, **10** credits well passed, **13** credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam. You will get, from home assignments and lab reports, max **3** credits if exam score is **< 4** and max **2** credits if exam score is **> 4**.

1. **SHORT PROBLEMS (3 credits):**

1.1 Consider a short Josephson junction of length $L << \lambda_J$ in the z direction and an external magnetic field applied in the plane of the junction in the x direction. Draw the spatial variation of the Josephson current as a function of z if the applied external flux is $2\Phi_0$ and $(5/2)\Phi_0$, where Φ_0 is the flux quanta. **(0.5 credit)**

1.2 On what length scale does microwave radiation penetrate a bulk superconductor and how does it depend on frequency? **(0.5 credit)**

1.3 Inductance of the RF SQUID loop is $2x10^{-9}$ H. What should be the critical current of the Josephson junction to provide non-hysteretic dependence of magnetic flux in the SQUID loop as a function of external applied flux? **(0.5 credit)**

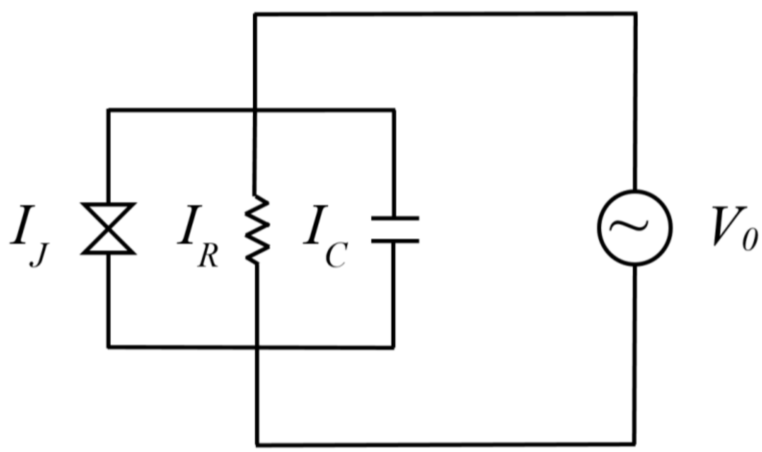
1.4 Draw the current voltage characteristic of an SIS Josephson junction. Describe the physical origin of the two branches and discuss the different currents across the SIS tunnel junction **(0.5 credit)**

1.5 Draw negative feedback flux-locked loop circuit for the dc SQUID and describe it's main components **(0.5 credit)**

1.6 Derive the expression for the Josephson inductance. Which is the expression of the energy stored in this non-linear inductor? **(0.5 credit)**

2. RSCJ model (3 credits)

2.1 Consider a constant voltage source V_0 connected across the Josephson junction. Using the RCSJ model, find the total current through the junction $I_{TOT}=I_C+I_J+I_R$ averaged over one time period Φ_0/V_0. Plot the I-V characteristic of such voltage biased junction. Does it depend on the quality factor of the Josephson junction? Motivate your answer. Draw the time dependent voltage across the junction for $I \cong I_C$ and $I >> I_C$, with I_C being the critical current of the junction. Consider junction with Q << 1. **(1.5 credits)**



2.2 A small area Josephson junction is biased with a constant current I_b which is much smaller than the critical current I_0 in the period of time $\infty < t \leq 0$. At $t = 0$ the bias current is reduced to zero. Consider you can model the junction with an RSCJ circuit and that you can linearize the differential equation describing the junction behavior $I_b \ll I_0$. Derive the dynamic of the junction at $\varphi\ (t)$ at $t \geq 0$ in the case of negligible losses ($Q = \infty$) demonstrating that the frequency of oscillation is the plasma frequency. Consider that the two arbitrary constants in differential equations must be determined in order to fully characterize the dynamic of the junction **(1.5 credits)**

3. dc SQUID (3 credits)

3.1 Consider two Josephson junctions in a superconducting ring (dc SQUID). Derive the relationship between two phase differences across these junctions and magnetic flux in the superconducting ring. **(1.5 credits)**

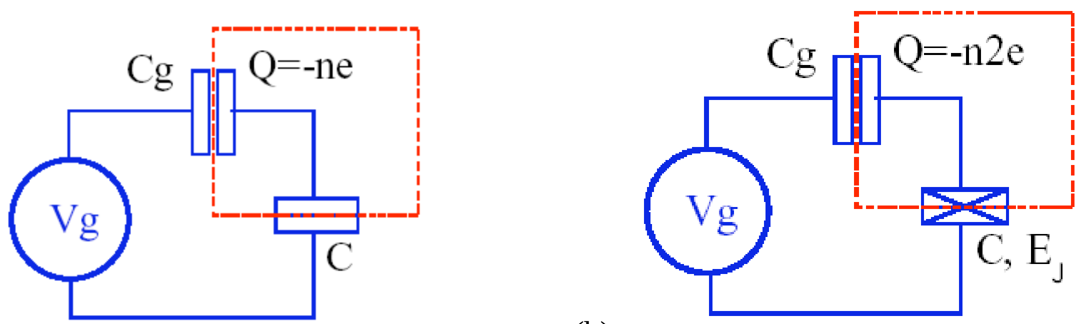
3.2 Derive magnetic flux dependence of the maximum critical current of the asymmetric DC SQUID with $I_{C,1} > I_{C,2}$ assuming negligible screening ($\beta_L << 1$). Show that the ratio of the maximum to minimum total critical current for the two-junction SQUID with negligible self-induced flux is given by $(\alpha + 1) / (\alpha - 1)$, where $\alpha = I_{C,1} / I_{C,2}$. **(1.5 credits)**

4. Single electron transistor (3 credits)

Figure below shows the schematic of a single electron box SEB (a) and of a single Cooper–pair box SCB (b).

4.1 For both physical systems, draw and comment the diagram of the energy of the box as a function of the gate voltage Vg, and for different numbers, n, of the charges on the box (omit the terms not dependent on n). Discuss the main differences between the two energy diagrams. **(1.5 credits)**

4.2 For both physical systems, draw and comment the dependence of the charge on the box as a function of the gate voltage (i.e. Coulomb staircase). Discuss what determines the smearing of the staircase for a SEB and a SCB. **(1.5 credits)**



(a) (b)

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5. Inductance and conductor losses (3 credits)

5.1 Consider a superconducting strip of width w< λ_L and thickness t< λ_L, where λ_L is the London penetration depth. Derive the inductance per unit length using the expression of the surface impedance of thin (t< λ_L) superconducting films:

$$Z = \frac{n_n}{n}\sigma_n(\omega\mu_0)^2\frac{\lambda_L^4}{t} + i\omega\mu_0\frac{\lambda_L^2}{t},$$

with n_n/n the fraction of unpaired electrons, σ_n the Drude conductivity, μ_0 the vacuum permeability, and ω the angular frequency. **(1.5 credits)**

5.2 What thickness of an aluminum superconducting strip is needed to minimize conductor losses at angular frequency $\omega = 2\pi \cdot 5$ GHz assuming that the temperature is much smaller than the superconducting transition temperature? **(1.5 credits)**