

Examination in MCC015: Superconducting Devices – Fundamentals and Applications

Wednesday June 1st 2022, 14.00 – 18.00

ML11 – Maskinteknik

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi 031 772 3318

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

You have to answer all problems

Total credits: **15**: ≥ 7 credits passed, ≥ 10 credits well passed, ≥ 13 credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam.

You will get, from all home assignments and lab reports, max **3** credits if exam score is < 5 and max **2** credits if exam score is ≥ 5 .

1. SHORT PROBLEMS (3 credits):

1.1 Consider a Josephson junction with $R_N = 500 \Omega$, $C = 1 \text{ pF}$ and $I_C = 1 \mu\text{A}$. Imagine placing an external resistor R_e in parallel to a Josephson junction with these parameters. Which is the value of R_e for the I-V characteristic junction to become non hysteretic? **(0.5 credit)**

1.2 A dc SQUID with a voltage modulation depth of $\Delta V_{pp} = 30 \mu\text{V}$ is included into the flux-locked loop with negative feedback. The feedback loop resistance is $R_f = 1 \text{ k}\Omega$ and mutual inductance is $M_f = 1 \text{ nH}$. Calculate external magnetic flux that can be applied to produce $1/8\Phi_0$ in the SQUID loop. The voltage gain of the preamplifier is $G_{LNA} = 4000$ **(0.5 credit)**

1.3 Derive an expression for the Josephson energy for the SNS junction with high transparency where the current-phase relationship is a sum of two components: $I(\phi) = I_{C1} \sin(\phi) + I_{C2} \sin(2\phi)$ **(0.5 credit)**

1.4 On what length scale does microwave radiation penetrate a bulk superconductor and how does it depend on frequency? **(0.5 credit)**

1.5 Please describe the detrimental effects on the performance of solid-state-based quantum bits coming from:

- a. Two-level systems
- b. Spurious spins

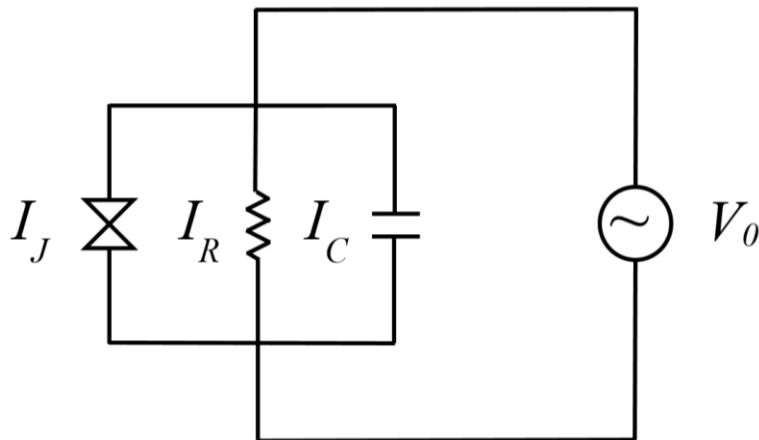
(0.5 credit)

1.6 Explain the main difference between a transmon qubit and a flux qubit, what are the advantages and disadvantages of the two? **(0.5 credit)**

2. Josephson Effect (3 credits)

2.1 RSCJ model.

Consider a constant voltage source V_0 connected across the Josephson junction. Using the RCSJ model, find the total current through the junction $I_{TOT}=I_C+I_J+I_R$ averaged over one time period Φ_0/V_0 . Plot the I-V characteristic of such voltage biased junction. Does it depend on the quality factor of the Josephson junction? Motivate your answer. Draw the time dependent voltage across the junction for $I \approx I_C$ and $I \gg I_C$, with I_C being the critical current of the junction. Consider a junction with $Q \ll 1$. (1.5 credits)

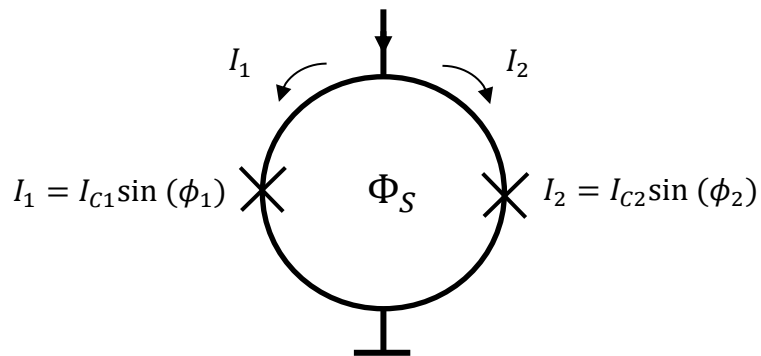


2.2 Magnetic field in the Josephson junction

Derive the magnetic field dependence of the critical current in a short Josephson junction with current-phase relationship given by the following expression: $I(\phi) = I_C \sin(2\phi)$. Assume the junction has a rectangular shape with length L and width, w . Plot the Fraunhofer pattern. (use trigonometric identity: $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$) (1.5 credits)

3. dc SQUID (3 credits)

Consider a dc SQUID with two Josephson junctions, see figure below. Derive an expression for the gauge invariant phase difference between junctions, $\phi_1 - \phi_2$ as a function of magnetic flux in the SQUID loop, Φ_S , where ϕ_1 and ϕ_2 are phase differences across junction 1 and 2, correspondingly. Neglect loop inductance.



4. Microwave properties of superconductors (3 credits)

The surface impedance $Z_S = R_S + iX_S$ is defined as $Z_S = \sqrt{\frac{i\omega\mu_0}{\sigma}}$, with ω the angular frequency, μ_0 the vacuum permeability, and $\sigma = \sigma_1 - i\sigma_2$ the complex conductivity.

4.1 Which properties of a conductor do the surface resistance R_S and surface reactance X_S describe?

For low enough temperatures and frequencies, the surface impedance of a superconductor can be approximated by

$$Z_S = \sqrt{\frac{\omega\mu_0}{\sigma_2}} \left(i + \frac{\sigma_1}{2\sigma_2} \right)$$

with $\sigma_1 = \frac{n_n}{n} \sigma_0$ and $\sigma_2 = \frac{1}{\omega\mu_0\lambda^2}$. Here $\frac{n_n}{n}$ is the fraction of unpaired electrons, σ_0 the Drude conductivity, and λ the London penetration depth. **(1.5 credits)**

4.2 Calculate the surface resistance and sketch R_S as a function of frequency (double log plot). **(1.5 credits)**

5. Superconducting qubits (3 credits)

A Transmon qubit has a frequency of 5GHz and E_J/E_C ratio of 50.

- a) Calculate the anharmonicity of the Transmon.
- b) How short can a π -pulse be before there is a significant risk of exciting the $|2\rangle$ state? Explain why
- c) How large is the frequency difference between state $|2\rangle$ and $|3\rangle$