Examination in MCC015: Superconducting Devices – Fundamentals and Applications

Wednesday May 31st 2023, 14.00 – 18.00

Johanneberg SB-L516

Responsible teachers: Alexei Kalaboukhov 073-7084195, Floriana Lombardi 031 772 3318

Allowed material: Your choice of calculator and a handwritten A4 single page with your own notes.

You have to answer all problems

Total credits: **15**: **≥ 7** credits passed, **≥ 10** credits well passed, **≥ 13** credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam. You will get, from all home assignments and lab reports, max **3** credits if exam score is **< 5** and max **2** credits if exam score is ≥ **5**.

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1. **SHORT PROBLEMS (3 credits):**

1.1 Consider a Josephson junction with R_N = 500 Ω, C = 1pF and I_C = 1 μA. Imagine placing an external resistor R_e in parallel to a Josephson junction with these parameters. Which is the value of R_e for the I-V characteristic junction to become non hysteretic? **(0.5 credit)**

1.2 Consider a dc SQUID with negligible inductance and identical Josephson junctions $I_{C1} = I_{C2}$. Draw the maximum critical current through the dc SQUID as a function of the normalized externally applied magnetic flux Φ_e/Φ_0, where $\Phi_0 = h/2e$ is the single flux quantum. How does the maximum critical current of the dc SQUID depend on the inductance? **(0.5 credit)**

1.3 Derive an expression for the Josephson energy for the SNS junction with high transparency where the current-phase relationship is a sum of two components: $I(\phi) = I_{C1}\sin(\phi) + I_{C2}\sin(2\phi)$ **(0.5 credit)**

1.4 On what length scale does microwave radiation penetrate a bulk superconductor and how does it depend on frequency? **(0.5 credit)**

1.5 Describe the detrimental effects on the performance of solid-state-based quantum bits coming from:

a. Two-level systems

b. Spurious spins

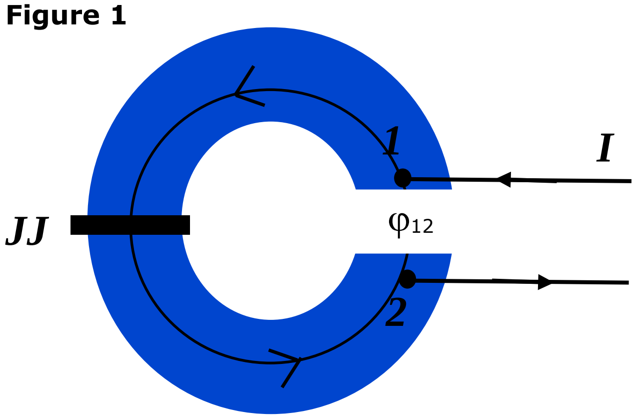
(0.5 credit)

1.6 Explain the main difference between a transmon qubit and a flux qubit, what are the advantages and disadvantages of the two? **(0.5 credit)**

2. Josephson junction in the ring (3 credits):

Consider a Josephson junction with a critical current I_C =1 μA incorporated in a superconducting ring of inductance L = 1 nH (see Figure 1). On the side opposite to the junction, a small piece is cut out, as illustrated in the figure. An external current source, connected across the opening, supplied a current I = 0.4 μA. Find the phase difference φ_{12} of the macroscopic wave function between the points 1 and 2.

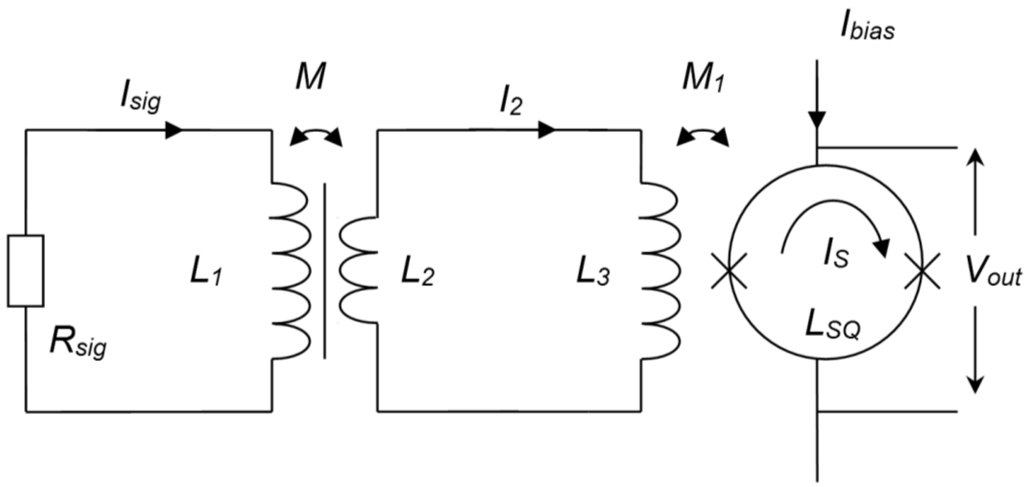
Figure 1



3. dc SQUID (3 credits)

DC-SQUID as a current meter.

Dc SQUID can be used to measure changes in various physical quantities that can be converted into magnetic flux in the SQUID loop. For electrical current measurements, a double superconducting transformer scheme is typically used as shown in the figure below. A current signal source is inductively connected to a superconducting flux transformer with $M = \alpha\sqrt{L_1 L_2}$, which in turn is also inductively coupled to the SQUID with $M_1 = \alpha_1\sqrt{L_2 L_{SQ}}$:



Derive the maximum current sensitivity (in A/Φ_0) of such double-transformer. Neglect resistive losses in the input transformer (R_{SIG} = 0) and assume that the coupling efficiencies α and α_1 ~1. How high should be the inductance of the input transformer L_1 to obtain current noise level below 100 fA/Hz$^{1/2}$ (1 fA = 10^{-15} A) assuming SQUID inductance $L_{SQ} = 10^{-10}$ H and SQUID flux noise $S_\Phi^{1/2} = 10^{-6}$ Φ_0/Hz$^{1/2}$?

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Department of Microtechnology and Nanoscience
SE-412 96 Göteborg, SWEDEN
Visiting address: Kemivägen 9

4. Microwave properties of superconductors (3 credits)

The surface impedance $Z_S = R_S + iX_S$ is defined as $Z_S = \sqrt{\frac{i\omega\mu_0}{\sigma}}$, with ω the angular frequency, $\mu_0$the vacuum permeability, and $\sigma = \sigma_1 - i\sigma_2$ the complex conductivity.

4.1 Which properties of a conductor do the surface resistance R_S and surface reactance X_S describe?

For low enough temperatures and frequencies, the surface impedance of a superconductor can be approximated by

$$Z_S = \sqrt{\frac{\omega\mu_0}{\sigma_2}}\left(i + \frac{\sigma_1}{2\sigma_2}\right)$$

with $\sigma_1 = \frac{n_n}{n}\sigma_0$ and $\sigma_2 = \frac{1}{\omega\mu_0\lambda^2}$. Here $\frac{n_n}{n}$ is the fraction of unpaired electrons, σ_0 the Drude conductivity, and λ the London penetration depth. **(1.5 credits)**

4.2 Calculate the surface resistance and sketch R_S as a function of frequency (double log plot). **(1.5 credits)**

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5. Superconducting qubits (3 credits)

5.1 Describe the advantages of a transmon qubit compared to a Cooper pair box (**1 credit**)

5.2 Describe different processes that can cause decoherence in superconducting qubits. What is the dominating source of decoherence for standard transmon qubits. Describe the dominating decoherence process. (**1 credit**)

5.3 What is the importance of the anharmonicity of superconducting qubits? (**1 credit**)

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