Examination in MCC015: Superconducting Devices – Fundamentals and Applications

Wednesday May 29th 2024, 14.00 – 18.00

Johanneberg SB-L216

Responsible teachers: Alexei Kalaboukhov: 073-7084195, Floriana Lombardi: 031 772 3318

Allowed material: Your choice of calculator and a handwritten A4 page with your own notes.

You have to answer all problems

Total credits: **15**: ≥ **7** credits passed, ≥ **10** credits well passed, ≥ **13** credits excellent.

All home assignments and lab reports will be valued and can be used in the evaluation of the exam. You will get, from all home assignments and lab reports, max **3** credits if exam score is < **5** and max **2** credits if exam score is ≥ **5**.

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1. **SHORT PROBLEMS (3 credits):**

1.1 Draw the *I-V* characteristic of the Josephson junction for Q<<1 (strong damping). By considering the mechanic analog of the junction draw the time dependent voltage across the junction for $I \approx I_C$ and $I >> I_C$, with I_C being the critical current of the junction. **(0.5 credit)**

1.2 Derive the inductance of the Josephson junction as a function of phase difference. **(0.5 credit)**

1.3 Derive an expression for the Josephson energy for the SNS junction with high transparency where the current-phase relationship is a sum of two components: $I(\phi) = I_{C1}\sin(\phi) + I_{C2}\sin(2\phi)$ **(0.5 credit)**

1.4 On what length scale does microwave radiation penetrate a bulk superconductor and how does it depend on frequency? **(0.5 credit)**

1.5 A dc SQUID with voltage modulation depth of ΔV_{pp}=40 μV is included into the flux-locked loop with negative feedback. The feedback loop resistance is R_f= 1 kΩ and mutual inductance is M_f = 1 nH. Calculate external magnetic flux that can be applied to produce $1/8\Phi_0$ in the SQUID loop. Voltage gain of the preamplifier is G_{LNA}=10000.

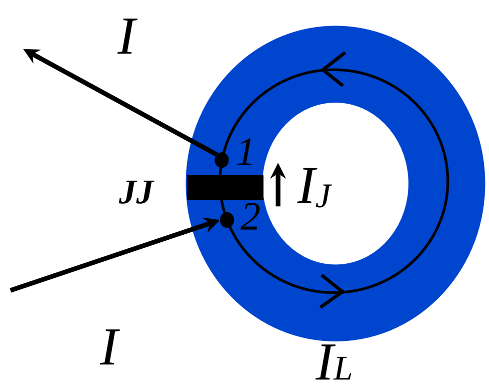
(0.5 credit)

1.6 Describe the detrimental effects on the performance of solid-state-based quantum bits coming from: a. Two-level systems b. Spurious spins **(0.5 credit)**

2. RF SQUID (3 cretids)

Consider an SNS Josephson junction with a critical current I_C = 1 μA incorporated in a superconducting ring with inductance L = 1 nH (see Figure 1). An external source supplies a current I, a part of which (I_L) passes through the ring and a part (I_J) through the junction. Calculate the value of the total current I supplied by the source if the current through the junction I_J is equal to I_C.

Figure 1



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3. RSJ model of the Josephson junction (3 credits)

3.1 A small area Josephson junction is biased with a constant current I_B which is much less that the critical current I_C in the period of time $-\infty < t \leq 0$. At $t = 0$ the bias current is reduced to 0. Consider you can model the junction with an RSCJ circuit and that you can linearize the differential equation describing the junction behavior assuming $I_B \ll I_C$. Derive the dynamic of the junction $\varphi\ (t)$ at $t \geq 0$ in the case of a small capacitance $Q \ll 1$. **(1.5 credits)**

3.2 Consider a constant voltage source V_0 connected across the Josephson junction. Using the RCSJ model, find the total current through the junction $I_{TOT} = I_C + I_J + I_R$ averaged over one time period Φ_0/V_0. Plot the I-V characteristic of such voltage biased junction. Does it depend on the quality factor of the Josephson junction? Motivate your answer. **(1.5 credits)**

4. Free energy of the rf SQUID (3 credits)

The total free energy of the superconducting loop with one Josephson junction (rf-SQUID) consists of two contributions from the Josephson and magnetic energy parts. Derive the total free energy of the rf SQUID in the absence of externally applied magnetic flux as a function of total current in the loop and make a schematic plot of this dependence. Show that for $\beta_{L,rf} < 1$ there is only one local minima at zero current. How many local minima corresponding to metastable energy states are at $\beta_{L,rf} = 30$?

5. Microwave properties of superconductors (3 credits)

Consider the superconducting half wave coplanar wave guide resonator of length d depicted in Fig. 2(a) connected at the two ends to transmission lines of impedance 50 Ohm via the coupling capacitors C_1 and C_2.
The input impedance of a transmission line terminated with a load Z_L at a distance d is given by

$$Z_{TL} = Z_0 \frac{Z_L + Z_0 \tanh \gamma d}{Z_0 + Z_L \tanh \gamma d},$$

with the characteristic wave impedance for small losses $Z_0 \approx \sqrt{\frac{L_l}{C_l}}$, with L_l and C_l the inductance and capacitance per unit length of the transmission line.
For an open-circuited transmission line resonator $Z_L \to \infty$ the above expression simplifies to

$$Z_{TL} = Z_0 \coth \gamma d$$

The wave propagation constant is given by $\gamma = \alpha + i\beta$, where α is the attenuation constant and the phase constant for small losses can be written as $\beta \approx \omega\sqrt{L_l C_l}$, with ω the angular frequency.

1) Which material parameters of the superconductor enter the attenuation constant α and phase constant β? (**0.5 credits**)
2) What is the value of the product of the phase constant and the length of the half-wave resonator, βd, at the resonance condition $\omega = \omega_r$ of the lowest fundamental mode? Illustrate your answer by sketching the voltage along the transmission line. (**0.5 credits**)
3) Close to the resonance frequency $\omega = \omega_r + \Delta\omega$ we can write for the input impedance of the half wave resonator:

$$Z_{TL} = \frac{Z_0}{\alpha d + i\frac{\pi\Delta\omega}{\omega_r}} = \frac{Z_0}{\alpha d + i\Delta\omega\sqrt{L_l C_l}}$$

The equivalent lumped element LCR circuit show in Fig. 2b close to resonance frequency $\omega = \omega_0 + \Delta\omega = \frac{1}{\sqrt{LC}} + \Delta\omega$ given by

$$Z_{LCR} = \frac{R}{1 + 2iRC\Delta\omega}$$

Mapping those two impedance expressions find the expressions of the equivalent resistor, R, capacitor C, and inductance L. (**2 credits**)

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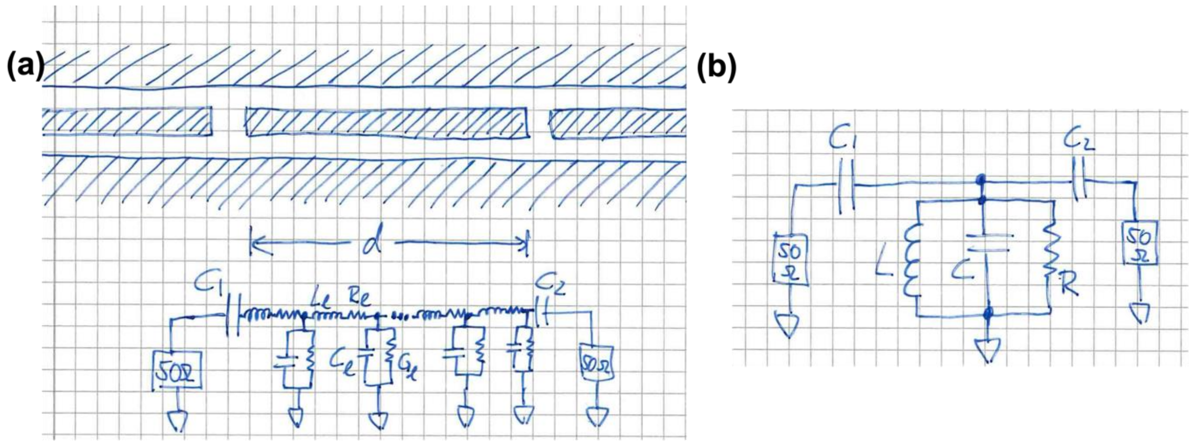

Figure 2. (a) The top panel is a top view of a coplanar waveguide half-wave resonator with the hatched regions being the superconducting film. The bottom panel is the equivalent transmission line circuit. (b) Equivalent lumped elementcircuit of the half-wave resonator close to resonance frequency.