

Lösning

MVE335 Matematik 1 för SI 2024-10-31

1. (a)

$$\frac{(x+h)^2 - (x-h)^2}{2h} = \frac{x^2 + h^2 + 2xh - (x^2 + h^2 - 2xh)}{2h}$$

$$= \frac{4xh}{2h} = 2x, \text{ för } h \neq 0$$

(b)

$$\frac{3(5-2R) - (3-4R)}{3} = 8 \quad \frac{15-6R-3+4R-24}{3} = 0 \quad -12-2R=0, R=-6$$

(c)

$$\frac{2x - x(1-2x) - 2(1-2x)}{2(1-2x)} = 0 \quad \frac{2x^2 + 5x - 2}{2(1-2x)} = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4 \cdot 2 \cdot (-2)}}{2 \cdot 2} \quad x = \frac{-5 \pm \sqrt{41}}{4}, (x \neq 1/2)$$

(d)

$$\frac{(x+1)(x+2) - 2 - 10x(x+2)}{x(x+2)} = 0$$

$$\frac{x^2 + 3x - 10x^2 - 20x}{x(x+2)} = 0$$

$$\frac{x(x+3-10x-20)}{x(x+2)} = 0$$

$$\frac{-9x-17}{x+2} = 0 \quad x = -\frac{17}{9}, (x \neq -2)$$

2. (a)

$$y = -3(x - (-2)) - 2 = -3x - 8 \quad \text{Sätt in i } x + 4y = 10$$

$$x - 12x - 32 = 10 \quad -11x = 42 \quad x = -\frac{42}{11}, y = \frac{3 \cdot 42}{11} - 8$$

(b)

$$\begin{cases} 10 - 3x - y = 0 \\ 20 - 2x - 5y = 0 \end{cases}$$

$$20 - 2x - 5(10 - 3x) = 0 \quad -30 + 13x = 0$$

$$x = \frac{30}{13}, y = 10 - \frac{90}{13}$$

(c)

$$y = x^3 + x = 6 - 4x^2 \quad x^3 + 4x^2 + x - 6 = (x+2)(x^2 + ax + b)$$

$$= x^3 + ax^2 + bx + 2x^2 + 2ax + 2b \quad \begin{cases} ax^2 + 2x^2 = 4x^2 \\ bx + 2ax = 1x \\ 2b = -6 \end{cases}$$

$$a = 2, b = -3, b + 2a = 1 \text{ OK!} \quad x^2 + 2x - 3 = 0 \quad x = \frac{-2 \pm \sqrt{4+12}}{2} \in \{1, -3\}$$

(d)

$$x^2 + y^2 + 2(3x - 5y) = 2 \quad (x+3)^2 + (y-5)^2 = 2 + 9 + 25 = 36 = 6^2$$

Punkt med störst x -koordinat = $(-3, 5) + (6, 0) = (3, 5)$

3. (a)

$$\begin{aligned} -3 \cos(v) - 2 &= 0 & x = \cos(v) &= -\frac{2}{3}, \quad y = \sin(v) \\ x^2 + y^2 &= 1 & \frac{4}{9} + y^2 &= 1, \quad y = \pm \frac{\sqrt{5}}{3} \\ 90^\circ < v < 180^\circ &\implies \sin(v) = \frac{\sqrt{5}}{3}, \quad \tan(v) = \frac{y}{x} = -\frac{\sqrt{5}}{2} \end{aligned}$$

(b)

$$\begin{aligned} A \cos(x - \varphi) &= \underbrace{A \cos \varphi}_3 \cdot \cos x + \underbrace{A \sin \varphi}_1 \cdot \sin x \\ A^2 &= (A \cos \varphi)^2 + (A \sin \varphi)^2 = 3^2 + 2^2 = 13, \quad A = \sqrt{13} \\ \tan \varphi &= \frac{A \sin \varphi}{A \cos \varphi} = \frac{-2}{-3}, \quad \varphi = \tan^{-1}\left(\frac{2}{3}\right) + \pi \approx 213.7^\circ \end{aligned}$$

(c)

$$\begin{aligned} \mathbf{F}_2 &= (x, y), \quad x^2 + y^2 = 3^2 \\ \mathbf{F}_1 &= (5, -2), \quad \mathbf{F}_1 + \mathbf{F}_2 = (5 + x, y - 2) \\ y - 2 &= 0 & x &= \pm \sqrt{9 - 4} = \pm \sqrt{5} \\ \cos v &= \frac{\mathbf{F}_1 \bullet \mathbf{F}_2}{|\mathbf{F}_1| |\mathbf{F}_2|} = \frac{5x - 2y}{3\sqrt{29}} \\ v &= \cos^{-1}\left(\frac{\pm 5\sqrt{5} - 4}{3\sqrt{29}}\right) \approx 63.6^\circ \text{ eller } 160^\circ \end{aligned}$$

(d)

$$\begin{aligned} A &= (0, -4) + t(-1, 4), \quad B = (4, 5) + s(-2, 3) & \begin{cases} -t = 4 - 2s \\ -4 + 4t = 5 + 3s \end{cases} \\ t &= 2s - 4 & -4 + 4(2s - 4) &= 5 + 3s & 5s &= 25 \\ s &= 5 \implies t = 6 & \text{dvs skärningspkt men ej kollision i } &(-6, 20) \end{aligned}$$

4. (a)

$$\begin{aligned} z = -2 + i &= \left\{ \begin{array}{l} r = \sqrt{4 + 1} = \sqrt{5} \\ \varphi = \tan^{-1}\left(\frac{1}{-2}\right) + \pi \end{array} \right\} \approx \sqrt{5}(\cos 153^\circ + i \sin 153^\circ) \\ w = \sqrt{7} - i\sqrt{2} &= \left\{ \begin{array}{l} r = \sqrt{7 + 2} = 3 \\ \varphi = \tan^{-1}\left(\frac{-\sqrt{2}}{1}\right) \end{array} \right\} \approx 3(\cos(-55^\circ) + i \sin(-55^\circ)) \end{aligned}$$

(b)

$$\begin{aligned} y &= \frac{fa - (b + c)d}{fc - (b + c)(e + f)} = \frac{(3i - 2)2 - (1 + 2i)(-2i)}{(3i - 2)i - (1 + 2i)(3i + 3)} \\ &= \frac{8i - 8}{-11i} = \frac{\text{pol}}{\text{pol}} = \frac{8\sqrt{2}e^{i135^\circ}}{11e^{-i90^\circ}} \\ &\approx 1.03e^{i225^\circ} \end{aligned}$$

(c)

$$\begin{aligned} 3(c - dz) - 4 \cdot (b - az) &= 0 & 2c - 3dz - 4b + 4az &= 0 \\ 3c - 4b &= (3d - 4a)z & z &= \frac{3c - 4b}{3d - 4a} \\ z &= \frac{6 - 4(2 + 1i)}{-9i - 8i \sin(75^\circ) - 8 \cos(75^\circ)} = \frac{-2 - 4i}{-2.0706 - 16.7274i} \\ &= \frac{\text{pol}}{\text{pol}} = \frac{4.47e^{-116.6^\circ i}}{16.86e^{-97.1^\circ i}} = 0.265e^{-19.5^\circ i} \end{aligned}$$

(d)

$$\begin{aligned}f'(t) &= i5Ae^{i(5t+\varphi)} \\i50Ae^{i\varphi}e^{i5t} + 50Ae^{i\varphi}e^{i5t} &= 75e^{i5t} \\Ae^{i\varphi}(i50 + 50) &= 75 \\Ae^{i\varphi} &= \frac{75}{50 + 50i} = \frac{\text{pol}}{\text{pol}} = \frac{75e^{0^\circ i}}{50\sqrt{2}e^{45^\circ i}} = 1.06e^{-45^\circ i}\end{aligned}$$

5. (a)

$$\begin{aligned}z^2 &= w \quad w^2 - 2iw - 4 = 0 \\w &= i \pm \sqrt{i^2 + 4} = i \pm \sqrt{3} = 2e^{i(\pi/6+n2\pi)} \text{ eller } = 2e^{i(5\pi/6+n2\pi)} \\z_{1,2} &= w^{1/2} = \sqrt{2}e^{i(\pi/12+n\pi)}, \quad n = 0, 1 \quad z_{3,4} = \sqrt{2}e^{i(5\pi/12+n\pi)}, \quad n = 0, 1\end{aligned}$$

(b)

$$\begin{aligned}\cos^2 x + \sin^2 x - 2 \sin x \cos x &= \cos 2x - \sin 2x \\1 - \sin 2x &= \cos 2x - \sin 2x \\\cos 2x &= 1 \iff 2x = n2\pi\end{aligned}$$

alternativt

$$\begin{aligned}\cos^2 x + \sin^2 x &= \cos^2 x - \sin^2 x \iff \sin^2 x = 0 \\x &= n\pi, n = 0, \pm 1, \pm 2, \dots\end{aligned}$$

6. Om sidan i en liksidig triangel är s så är höjden $= \sqrt{s^2 - (s/2)^2} = s\sqrt{3}/2$. Därför fås koordinaterna för tredje hörnet genom att utgå från mittpunkten på den givna sidan och gå vinkelrätt mot denna så långt som höjdens längd. Mittpunkten är $(1, -1)$ och riktning skall vara vinkelrät mot sidan $(6, -4)$, tex $(4, 6)$. Då får koord för tredje punkten: $(1, -1) \pm |(6, -4)| \frac{\sqrt{3}}{2} (1/|(4, 6)|)(4, 6) = (1, -1) \pm \sqrt{3}(2, 3)$.

Liksidiig triangel med hörn på cirkel med radie r har vinkel $2\pi/3$ mellan vektorerna från cirkelns centrum till triangels hörn. Cosinussatsen ger då att $s^2 = r^2 + r^2 - 2r \cdot r \cos(2\pi/3) = 3r^2$, dvs $r = s/\sqrt{3} = 2\sqrt{13}/3$.

7. Skärningspkt mellan medianerna från A resp. B:

$$(3, -2) + t((7/2, 3/2) - (3, -2)) = (7, 3) + s((3/2, -1) - (7, 3))$$

$$\begin{cases} 3 + t/2 = 7 - 11s/2 \\ -2 + 7t/2 = 3 - 4s \end{cases}$$

$$-23 = -46 + 69s/2, \quad s = 46/69 = 2/3$$

$$\text{Medianernas skärningspunkt} = (7, 3) + \frac{2}{3}(-\frac{11}{2}, -4)$$

Skärningspunkt mellan höjderna från A resp. B:

$$(3, -2) + t(3, -7) = (7, 3) + s(2, 3)$$

$$\begin{cases} 3 + 3t = 7 + 2s \\ -2 - 7t/2 = 3 + 3s \end{cases}$$

$$-13 - 23t = -15, \quad t = 2/23$$

$$\text{Höjdernas skärningspunkt} = (3, -2) + \frac{2}{23}(3, -7)$$