

Lösning
MVE335 Matematik 1 för SI 2025-01-09

1. (a)

$$\frac{(-2+h)^2 - (-2-h)^2}{2h} = \frac{4+h^2-4h - (4+h^2+4h)}{2h}$$

$$= \frac{-8h}{2h} = -4, \text{ för } h \neq 0$$

(b)

$$\frac{8-10R-(7-20R)}{5} = 4 \quad \frac{40-50R-7+20R-20}{5} = 0 \quad 13-30R=0, \quad R=13/30$$

(c)

$$\frac{x(1+2x)-1}{1+2x} = 1 \quad \frac{x(1+2x)-1-(1+2x)}{1+2x} = 0$$

$$2x^2 - x - 2 = 0 \quad x = \frac{1 \pm \sqrt{17}}{4}, \quad (x \neq -1/2)$$

(d)

$$\frac{(x+2)(x-2)+4-10x(x-2)}{x(x-2)} = 0$$

$$\frac{-9x^2+20x}{x(x-2)} = 0$$

$$\frac{-9x+20}{x-2} = 0$$

$$x = \frac{20}{9}, \quad (x \neq 2, x \neq 0)$$

2. (a)

$$y = -2(x-1) - 3 = -2x - 1 \quad \text{Sätt in i } 4x + 5y = 6$$

$$4x - 10x - 5 = 6 \quad -6x = 11 \quad x = -\frac{11}{6}, \quad y = \frac{11}{3} - 1$$

(b)

$$\begin{cases} 3-3x-2y=0 \\ 4-3x-5y=0 \end{cases}$$

$$3-2y=4-5y \quad -1+3y=0$$

$$x=1-\frac{2}{9}, \quad y=\frac{1}{3}$$

(c)

$$y = x^3 + 3x^2 + 3x - 3 = 10x^2 - 10x \quad x^3 - 7x^2 + 13x - 3 = (x-3)(x^2 + ax + b)$$

$$= x^3 + ax^2 + bx - 3x^2 - 3ax - 3b \quad \begin{cases} ax^2 - 3x^2 = -7x^2 \\ bx - 3ax = 13x \\ -3b = -3 \end{cases}$$

$$a = -4, \quad b = 1, \quad b - 3a = 13 \text{ OK!} \quad x^2 - 4x + 1 = 0 \quad x = 2 \pm \sqrt{3}$$

(d)

$$x^2 + y^2 + 4(x-2y) = 16 \quad (x+2)^2 + (y-4)^2 = 16 + 4 + 16 = 36 = 6^2$$

$$\text{Kvadratens övre vänstra hörn} = (-2, 4) + (-6, 6) = (-8, 10)$$

3. (a)

$$\begin{aligned}\cos(v) - 2\sin(v) &= 0 & x &= \cos(v), \quad y = \sin(v) \\ x^2 + y^2 &= 1 & (2y)^2 + y^2 &= 1, \quad y = \pm \frac{1}{\sqrt{5}} \\ 0^\circ < v < 90^\circ &\implies \sin(v) = \frac{1}{\sqrt{5}}, \quad \cos(v) = \frac{2}{\sqrt{5}}\end{aligned}$$

(b)

$$\begin{aligned}A \cos(x - \varphi) &= \underbrace{A \cos \varphi}_1 \cdot \cos x + \underbrace{A \sin \varphi}_{-2} \cdot \sin x \\ A^2 &= (A \cos \varphi)^2 + (A \sin \varphi)^2 = 1^2 + 2^2 = 5, \quad A = \sqrt{5} \\ \tan \varphi &= \frac{A \sin \varphi}{A \cos \varphi} = \frac{-2}{1}, \quad \varphi = \tan^{-1}(-2) \approx -63^\circ\end{aligned}$$

(c)

$$\begin{aligned}\mathbf{F}_2 &= (x, y), \quad x^2 + y^2 = 6^2 \\ \mathbf{F}_1 &= (-2, -2), \quad \mathbf{F}_1 + \mathbf{F}_2 = (-2 + x, y - 2) \\ -2 + x &= 0 \quad y = \pm \sqrt{36 - 4} = \pm 4\sqrt{2} \\ \cos v &= \frac{\mathbf{F}_1 \bullet \mathbf{F}_2}{|\mathbf{F}_1||\mathbf{F}_2|} = \frac{-2x - 2y}{12\sqrt{2}} \\ v &= \cos^{-1}\left(\frac{-4 + \pm 8\sqrt{2}}{12\sqrt{2}}\right) \approx 64.5^\circ \text{ eller } 154.5^\circ\end{aligned}$$

(d)

$$\begin{aligned}A &= (0, 8) + t(-4, 3), \quad B = (5, -7) + s\mathbf{v} \\ (0, 8) + 5(-4, 3) &= (5, -7) + 5\mathbf{v} \\ (0, 8) + 5(-4, 3) - (5, -7) &= 5\mathbf{v} \\ (-25, 30) &= 5\mathbf{v} \quad (-5, 6) = \mathbf{v}\end{aligned}$$

4. (a)

$$\begin{aligned}z = -2 + i\sqrt{2} &= \left\{ \begin{array}{l} r = \sqrt{4+2} = \sqrt{6} \\ \varphi = \tan^{-1}\left(\frac{\sqrt{2}}{-2}\right) + \pi \end{array} \right\} \approx \sqrt{6}(\cos 145^\circ + i \sin 145^\circ) \\ w = 3 - i\sqrt{3} &= \left\{ \begin{array}{l} r = \sqrt{9+3} = 2\sqrt{3} \\ \varphi = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right) \end{array} \right\} = 2\sqrt{3}(\cos(-30^\circ) + i \sin(-30^\circ))\end{aligned}$$

(b)

$$\begin{aligned}y &= \frac{fa - (b+c)d}{fc - (b+c)(e+f)} = \frac{2 - (-2i)(3+i)}{2 - 3i - (-2i)(3+2i)} \\ &= \frac{6i}{-2+3i} = \frac{\text{pol}}{\text{pol}} = \frac{6e^{i90^\circ}}{\sqrt{13}e^{i124^\circ}} \\ &\approx 1.66e^{-i34^\circ}\end{aligned}$$

(c)

$$\begin{aligned}(c - dz) - 2(b - az) &= 0 & c - dz - 2b + 2az &= 0 \\ c - 2b &= (d - 2a)z & z &= \frac{c - 2b}{d - 2a} \\ z &= \frac{2 - 2(1-i)}{-3 + i - 4\cos(25^\circ) - 4i\sin(25^\circ)} = \frac{2i}{-4.96 \dots - i2.49 \dots} \\ &= \frac{\text{pol}}{\text{pol}} = \frac{2e^{90^\circ i}}{5.55 \dots e^{-153.4 \dots i}} = 0.36e^{-117^\circ i}\end{aligned}$$

(d)

$$\begin{aligned}f'(t) &= i2Ae^{i(2t+\varphi)} \\i6Ae^{i\varphi}e^{i2t} + 8Ae^{i\varphi}e^{i2t} &= 20e^{i2t} \\Ae^{i\varphi}(i6+8) &= 20 \\Ae^{i\varphi} &= \frac{20}{8+6i} = \frac{\text{pol}}{\text{pol}} = \frac{20e^{0^\circ i}}{10e^{37^\circ i}} = 2e^{-37^\circ i}\end{aligned}$$

5. (a)

$$\begin{aligned}z^2 &= w \quad w^2 - (2+2i)w + 2i = 0 \\w &= (1+i) \pm \sqrt{(1+i)^2 - 2i} = 1+i = \sqrt{2}e^{i(\pi/4+n2\pi)} \\z_{1,2} &= w^{1/2} = \sqrt[4]{2}e^{i(\pi/8)}, \quad z_{3,4} = \sqrt[4]{2}e^{i(\pi/8+\pi)}\end{aligned}$$

(b)

$$\begin{aligned}2\cos 2x \sin 2x &= \cos^2 2x - \sin^2 2x \\\sin 4x &= \cos 4x \quad \cos\left(\frac{\pi}{2} - 4x\right) = \cos(4x) \\\frac{\pi}{2} - 4x &= \pm 4x + n2\pi \\x &= \frac{\pi}{16} - \frac{n\pi}{4}\end{aligned}$$

6. Om likbenta sidan är $2AB$ så är höjden $h = \sqrt{(2AB)^2 - (AB/2)^2} = AB\sqrt{15}/2$. Mitt-punkten på AB är $M = (1, -1)$ och riktning $\mathbf{v}$ vinkelrät mot AB tex $\mathbf{v} = (4, 6)$. Koord för tredje punkten:

$$M \pm h \frac{\mathbf{v}}{|\mathbf{v}|} = (1, -1) \pm |(6, -4)|(\sqrt{15}/2)((4, 6)/|(4, 6)|) = (1, -1) \pm \sqrt{15}(2, 3)$$

7.

$$\begin{aligned}t^3 + \frac{1}{t^3} + 1.5 &= 0 \\t^3 &= w \quad w^2 + 1.5w + 1 = 0 \\w &= -3/4 \pm i\sqrt{7}/4 = e^{i(\pm\varphi+n2\pi)}, \quad \varphi = 138.6^\circ \\t_n &= e^{i(\varphi/3+n2\pi/3)}, \quad n = 0, 1, 2 \\x_n &= t_n + \bar{t}_n = 2\cos(\varphi/3 + n2\pi/3) \\x_0 &= 2 \cdot 0.6922 \quad x_1 = -2 \cdot 0.9711 \quad x_2 = 2 \cdot 0.279\end{aligned}$$