240110 SEE080 Mekanik, tenta 12 januari 2024

1

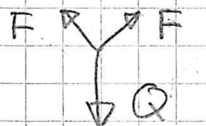
(1.) a) Den ekvivalenta kraften i B ger inget moment i B ⇒ det ekvivalenta momentet i B måste vara lika med de tre krafternas moment. $\overset{\curvearrowleft}{M}_B = -2\cdot2 + 1\cdot2 = -2$ Nm (alt. $-2\cdot2+\sqrt{2}\cdot\frac{2\sqrt{2}}{2} = -2$ Nm)

Svar: D

b) Vertikala kraften ger moment, motverkas av F_C.

Svar: C

c) Kraften i linan lika stor överallt. Jämvikt:



$$\uparrow \Big| \quad \frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} - Q = 0 \Rightarrow Q = \sqrt{2}F$$

Svar: D

d) Newtons 2:a lag: $-2kx - 2kx - 2kx = 3m\ddot{x}$

$$\Rightarrow \ddot{x} + \frac{6k}{3m}x = 0 \Rightarrow \ddot{x} + \frac{2k}{m} = 0 \Rightarrow \omega^2 = \frac{2k}{m}$$

$$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}} = \pi\sqrt{\frac{2m}{k}}$$

Svar: A

e) Svar: B (rörelsemängdsmomentet; Keplers 2:a lag)

f.) $m = 1$ kg, $v_0 = 1$ m/s, $v_1 = ?$

A) $F\cdot t = mv_1 - mv_0 \Rightarrow v_1 = 2$ m/s

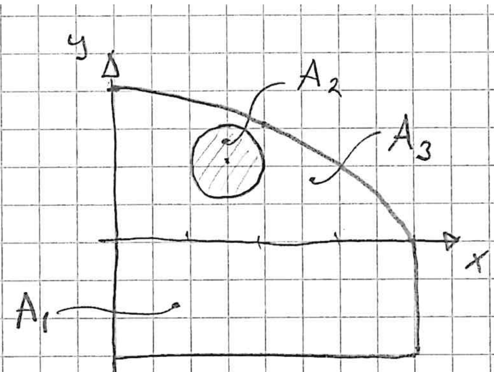
B) $\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = F\cdot s \Rightarrow v_1 = \sqrt{3}$ m/s $\approx 1{,}7$ m/s

C) Som B: $v_1 = \sqrt{3}$ m/s

D) Som A: $v_1 = 2$ m/s

E) $\frac{1}{2}mv_1^2 = 2\cdot\frac{1}{2}mv_0^2 \Rightarrow v_1 = \sqrt{2}v_0 \approx 1{,}4$ m/s

Svar: E



(2) $y = 4 - \frac{1}{16}x^2$

$\bar{x} = \frac{1}{A}\sum A_i \bar{x}_i \quad , \quad \bar{y} = \frac{1}{A} A_i \bar{y}_i$

$A_1 = 3 \cdot 8 = 24$

$A_2 = -\pi \cdot 1^2 = -\pi$ (hål)

$A_3 = \int_0^8 y\,dx = \int_0^8 \left(4 - \frac{1}{16}x^2\right)dx = \left[4x - \frac{1}{16}\cdot\frac{x^3}{3}\right]_0^8 = \frac{64}{3} \approx 21,33$

$A = A_1 + A_2 + A_3 = \frac{136}{3} - \pi \approx 42,19$

$\bar{x}_1 = 4 \quad , \quad \bar{x}_2 = 3$

$\bar{x}_3 = \frac{1}{A_3}\int x\,dA = \frac{1}{A_3}\int_0^8 x\,y\,dx = \frac{1}{A_3}\int_0^8 \left(4x - \frac{1}{16}x^3\right)dx =$

$= \frac{3}{64}\left[2x^2 - \frac{1}{16}\cdot\frac{x^4}{4}\right]_0^8 = 3$

$\bar{x} = \frac{1}{\frac{136}{3} - \pi}\left(24 \cdot 4 - \pi \cdot 3 + \frac{64}{3}\cdot 3\right) = \frac{1}{\frac{136}{3} - \pi}(160 - 3\pi) \approx \underline{\underline{3,57}}$

$\bar{y}_1 = -\frac{3}{2} \quad , \quad \bar{y}_2 = 2 \qquad \bar{y}_3$: två metoder

alt. 1: $\bar{y}_3 = \frac{1}{A_3}\int y\,dA = \frac{1}{A_3}\int y\,x\,dy = \left\{ y = 4 - \frac{x^2}{16} \Rightarrow x = 4\sqrt{4-y} \right\} =$

$= \frac{4}{A_3}\int_0^4 y \cdot \sqrt{4-y}\,dy = \frac{4}{A_3}\left\{\left[y\,\frac{(4-y)^{3/2}}{-3/2}\right]_0^4 - \int_0^4 \frac{(4-y)^{3/2}}{-3/2}\,dy\right\} =$

$= \frac{4}{A_3}\left\{0 + \frac{2}{3}\int_0^4 (4-y)^{3/2}\,dy\right\} = 4 \cdot \frac{3}{64}\cdot\frac{2}{3}\left[\frac{(4-y)^{5/2}}{-5/2}\right]_0^4 = \frac{8}{5} = 1,6$

alt. 2: $\bar{y}_3 = \frac{1}{A_3}\int_0^8 \frac{y}{2}\cdot y\,dx = \frac{1}{2A_3}\int_0^8 \left(4 - \frac{1}{16}x^2\right)^2 dx =$

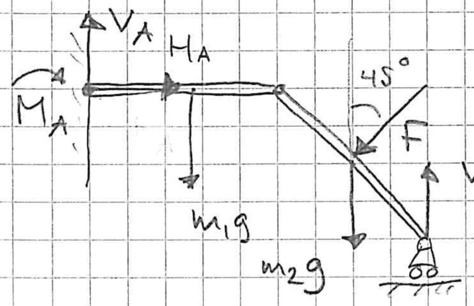
$= \frac{1}{2A_3}\int_0^8 \left(16 - \frac{1}{2}x^2 + \frac{1}{256}x^4\right)dx = \frac{3}{2\cdot 64}\left[16x - \frac{x^3}{6} + \frac{x^5}{5\cdot 256}\right]_0^8 = \frac{8}{5} = 1,6$

(2) forts

$$\bar{y} = \frac{1}{\frac{136}{3} - \pi} \left(24 \cdot \left(-\frac{3}{2}\right) - \pi \cdot 2 + \frac{64}{3} \cdot \frac{8}{5} \right) \approx \underline{\underline{-0{,}19}}$$

Svar: $(\bar{x}, \bar{y}) = (3{,}57; -0{,}19)$

(3)



Sökt: V_A, H_A, M_A, V_B

$m_1 = 15$ kg

$m_2 = 10$ kg

$F = 50$ N

Jämvikt (hela systemet):

$\uparrow |$ $V_A - m_1 g - m_2 g - \frac{1}{\sqrt{2}} F + V_B = 0 \Rightarrow V_A + V_B - 280{,}36 = 0$ (1)

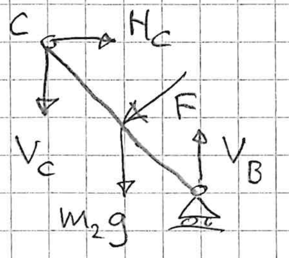
$\rightarrow |$ $H_A - \frac{1}{\sqrt{2}} F = 0 \Rightarrow H_A = \frac{1}{\sqrt{2}} F = 35{,}36$ N (2)

$\overset{\frown}{A} |$ $M_A + m_1 g \cdot 1 + m_2 g \cdot 3 + \frac{1}{\sqrt{2}} F \cdot 3 + \frac{1}{\sqrt{2}} F \cdot 1 - V_B \cdot 4 = 0$

$\Rightarrow M_A - 4 V_B + 582{,}42 = 0$ (3)

V_A, V_B & M_A söks, men bara två ekv, (1) & (3).

Frilägg den sneda stången:



$\overset{\frown}{C} |$ $F \cdot \sqrt{2} + m_2 g \cdot 1 - V_B \cdot 2 = 0$

$\Rightarrow V_B = \frac{1}{2}(F \cdot \sqrt{2} + m_2 g \cdot 1) = 84{,}36$ N

(1) $\Rightarrow V_A = 280{,}36 - V_B = 196{,}0$ N

(3) $\Rightarrow M_A = 4 V_B - 582{,}42 = -244{,}98$ Nm

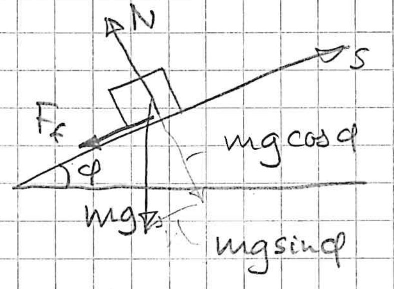
Svar: $V_A = 196$ N

$H_A = 35$ N

$M_A = -245$ Nm

$V_B = 84$ N

4.



$$N = mg\cos\varphi \qquad F_f = \mu N = \mu mg\cos\varphi \qquad \begin{cases} \varphi = 10^\circ \\ \mu = 0{,}2 \\ m = 2\text{ kg} \end{cases}$$

Kraften F, riktning ↙ : $F = mg\sin\varphi + \mu mg\cos\varphi$

Newtons 2:a lag: $m\ddot{s} = -F = -mg(\sin\varphi + \mu\cos\varphi) \Rightarrow$

$$\Rightarrow \ddot{s} = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} = -g(\sin\varphi + \mu\cos\varphi).$$

Separera, integrera: $\displaystyle\int_{v_0}^{0} v\,dv = \int_0^s -g(\sin\varphi + \mu\cos\varphi)\,ds$

$$\Rightarrow -\frac{1}{2}v_0^2 = -g(\sin\varphi + \mu\cos\varphi)\cdot s \Rightarrow s = \frac{\frac{1}{2}v_0^2}{g(\sin\varphi + \mu\cos\varphi)} = \underline{\underline{2{,}20\text{ m}}}$$

Alt: Rörelseenergin blir lägesenergi + värme från friktionskraftens arbete:

$$\frac{1}{2}mv_0^2 = mg\cdot s\cdot\sin\varphi + \mu mg\cos\varphi\cdot s \Rightarrow s = \text{som ovan}$$

Konstant acceleration, motriktad rörelsen:

$$a = -\frac{F}{m} = -g(\sin\varphi + \mu\cos\varphi) = -3{,}63\text{ m/s}^2$$

$$v = v_0 + at \Rightarrow t = \frac{v - v_0}{a} = \frac{0-4}{-3{,}63} = \underline{1{,}10\text{ s}}$$

Friktionskraften $F_f = \mu mg\cos\varphi = 3{,}86$ N (eller $< 3{,}86$ N)

Tyngdkraftens komponent ↙ : $mg\sin\varphi = 3{,}40\text{ N} < F_f$

$\Rightarrow$ Lådan glider inte ner.

Svar: $s = 2{,}20$ m, $t = 1{,}10$ s, lådan glider <u>inte ner</u>

5. $F(t) = kt$, $k = 27$ N/s. $m = 8$ kg. $x(0) = 0$. $v(0) = 0$

Newtons 2:a lag $m\frac{dv}{dt} = kt$. Separera, integrera:

$$\int_0^v m\,dv = \int_0^t kt\,dt \Rightarrow mv = \frac{1}{2}kt^2 \Rightarrow v = \frac{k}{2m}t^2 \quad (1)$$

$$\frac{dx}{dt} = v \Rightarrow \frac{dx}{dt} = \frac{k}{2m}t^2.$$ Separera, integrera:

$$\int_0^x dx = \int_0^t \frac{k}{2m}t^2\,dt \Rightarrow x = \frac{k}{6m}t^3 \quad (2)$$

$$(2) \Rightarrow t = \left(\frac{6m}{k}x\right)^{1/3} = \left(\frac{6\cdot 8}{27}\cdot 8\right)^{1/3} = 2{,}42 \text{ s.}$$

Insättning i (1): $v = \frac{k}{2m}\cdot\left(\frac{6m}{k}x\right)^{2/3} = \left(\frac{k}{m}\right)^{1/3}\cdot\frac{6^{2/3}}{2}\cdot x^{2/3} = \underline{\underline{9{,}9 \text{ m/s}}}$

Energiprincipen: $W = \frac{1}{2}mv^2 = 392$ Nm (J)

Alt: $$W = \int F\,dx = \int_0^8 k\cdot\left(\frac{6m}{k}x\right)^{1/3}dx = k^{2/3}(6m)^{1/3}\cdot\frac{8^{4/3}}{4/3} = \underline{\underline{392 \text{ Nm}}}$$

6. $r = 50$ m, $v_0 = 0$, $v_1 = 50$ km/h $= 13{,}89$ m/s. a_s = konst.

$s_1 = 2\pi r$ (ett varv). $s_1 = \frac{1}{2}a_s t_1^2$, $v_1 = a_s t_1 \Rightarrow$

$$2\pi r = \frac{1}{2}a_s\left(\frac{v_1}{a_s}\right)^2 \Rightarrow a_s = \frac{v_1^2}{4\pi r}.$$

$s_2 = 4\pi r$ (två varv): $4\pi r = \frac{1}{2}a_s t_2^2 = \frac{1}{2}\cdot\frac{v_1^2}{4\pi r}t_2^2 \Rightarrow$

$$\Rightarrow t_2 = \sqrt{2}\cdot\frac{4\pi r}{v_1}. \quad v_2 = a_s t_2 = \frac{v_1^2}{4\pi r}\cdot\sqrt{2}\,\frac{4\pi r}{v_1} = \sqrt{2}\,v_1$$

$F_f = \mu N = \mu mg$. Newtons 2:a lag: $F_f = ma \Rightarrow \mu g = a$

$$a = \sqrt{a_n^2 + a_s^2}. \quad a_n = \frac{v_2^2}{r} = \frac{2v_1^2}{r}. \quad a_s = \frac{v_1^2}{4\pi r}$$

$$\Rightarrow a = \frac{v_1^2}{r}\sqrt{4 + \frac{1}{16\pi^2}} = 7{,}722 \text{ m/s}$$

$$\mu g = a \Rightarrow \mu = \frac{a}{g} = \frac{7{,}722}{9{,}8} = \underline{\underline{0{,}79}}$$

7. $\tau = 12\ \text{h} = 12 \cdot 3600\ \text{s}$. $r_p = R + 1000\ \text{km} = 7\,380\ \text{km} = 7{,}38 \cdot 10^6\ \text{m}$

Keplers 3:e lag: $\frac{a^3}{\tau^2} = \frac{GM}{4\pi^2} \Rightarrow a = \left(\frac{GM}{4\pi^2}\tau^2\right)^{1/3} = 2{,}66 \cdot 10^7\ \text{m}$

$= 26\,600\ \text{km}$

$$r_p = \frac{a(1-e^2)}{1+e\cos 0^\circ} = a(1-e) \Rightarrow$$

$$\Rightarrow e = 1 - \frac{r_p}{a} = 1 - \frac{7380}{26\,600} = \underline{\underline{0{,}723}}$$

$$v_p^2 = GM\left(\frac{2}{r_p} - \frac{1}{a}\right) \Rightarrow v_p = 9\,641\ \text{m/s} = \underline{\underline{9{,}6\ \text{km/s}}}$$

$$r_a = \frac{a(1-e^2)}{1+e\cos 180^\circ} = a(1+e) = 45\,830\ \text{km}$$

Keplers 2:a lag: $r_p v_p = r_a v_a \Rightarrow v_a = \frac{r_p}{r_a} v_p = 1552\ \text{m/s}$

$\approx \underline{\underline{1{,}6\ \text{km/s}}}$

$E = T + V = \text{konst.}$

Välj någon punkt i banan, t. ex. r_p:

$$E = \frac{1}{2} m v_p^2 - \frac{GMm}{r_p} = \underline{\underline{-3{,}7 \cdot 10^9\ \text{J}}}$$