Discrete Event Systems

Course code: SSY165

Examination 2021-10-23

Time: 8:30-12:30, Location: M-building

Teacher: Bengt Lennartson, phone 3722

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on Wednesday *November 10* and Thursday *November 11*, 12:30-13:00 by Zoom.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Electrical Engineering
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1

Prove the following predicate equivalence

$$\forall x[P(x)] \to \forall x[Q(x)] \Leftrightarrow \forall x\, \exists y[P(y) \to Q(x)]$$

and give an intuitive explanation of this equivalence. (3 p)

2

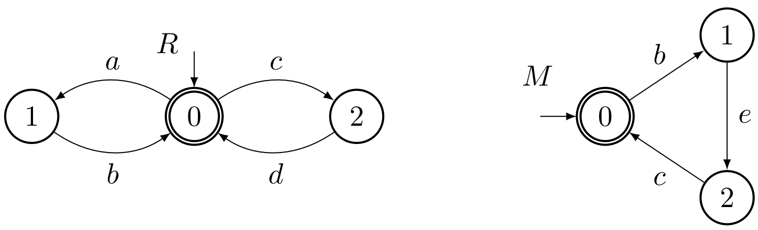
For a given universal set Ω, prove the following set expression implication

$$A \subseteq B \Rightarrow (\sim A \cup B) \cap (B \cup \sim A \setminus B) = \Omega$$

by only using equivalent set expressions. *Hint:* For the subset expression, use the fact that $A \subseteq B \Leftrightarrow A \cup B = B$ (3 p)

3

Consider the following automata models for a common resource R and a machine M.



a) For the resource R , give the formal language $\mathcal{L}(R)$ as well as the marked language $\mathcal{L}_m(R)$. (1 p)

b) Formulate a Petri net for the synchronized system $R \,\|\, M$, including the marking vector for the synchronized marked state (2 p)

c) Generate the automaton $R \,\|\, M$. (2 p)

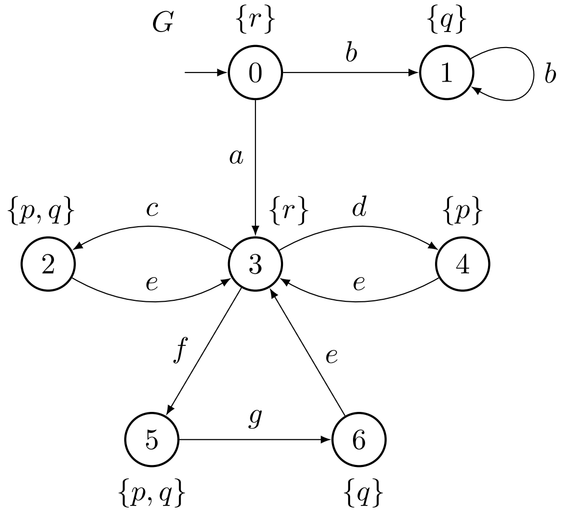
d) Add one or more arcs in the Petri net for $R \,\|\, M$ to avoid the blocking state. (1 p)

4

A temporal logic supervisor includes all those states in a transition system G that satisfy a temporal logic formula φ. Use μ-calculus to determine a temporal logic supervisor automaton for the plant G below which satisfies the formula

$$\varphi = \exists\Box(q \vee r) \wedge \exists\Diamond p.$$

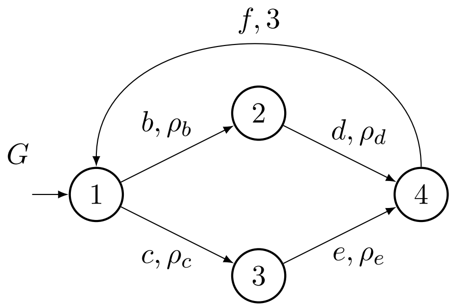
Thus, formulate an automaton that only includes those states in G where φ is valid. *Hint:* The conjunction can in this example be handled by solving two separate μ-calculus problems.



(5 p)

5

Consider the automaton G, including immediate rewards $\rho(x,a)$ on each transition from state x with action a.



Determine the relation between the rewards $\rho_b > 0$, $\rho_c > 0$, $\rho_d > 0$, and $\rho_e > 0$ for which the action b is the optimal choice that maximizes the total reward

$$\sum_{k=0}^{\infty} \gamma^k \rho(x_k, a_k)$$

for an arbitrary discounting factor $0 < \gamma < 1$. (4 p)

6

To reduce the number of states, local events that are not involved in any other subsystems can be replaced by the hidden event τ. Any τ transition can then be removed when no alternative transitions are involved in the source state of such τ transitions, and the source and target states have the same state label. Removing a τ transition means that the source and target states are merged into one state.

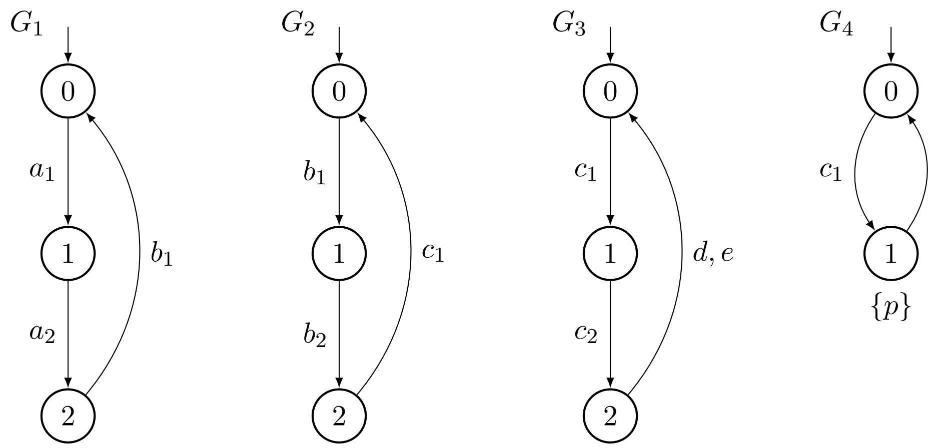
Apply this reduction principle on the following synchronized system

$$G_1 \,\|\, G_2 \,\|\, G_3 \,\|\, G_4,$$

where the individual transition systems are given below, and local τ transitions are identified and removed before every synchronization. Show that the final reduced system

$$((G_1^{\mathcal{A}} \,\|\, G_2^{\mathcal{A}})^{\mathcal{A}} \,\|\, G_3^{\mathcal{A}} \,\|\, G_4)^{\mathcal{A}}$$

only includes three states where one state is a blocking state. The reduction (abstraction operator) $\mathcal{A}$ on any automaton includes the replacement of local events with τ, followed by the state reduction.



(4 p)

Solution Exam DES 2021-10-23 BL

1. $\forall x\,[P(x)] \to \forall x\,[Q(x)] \Leftrightarrow \neg(P(a_1)\wedge\ldots\wedge P(a_n)) \vee (Q(a_1)\wedge\ldots\wedge Q(a_n))$

$\Leftrightarrow \neg P(a_1)\vee\ldots\vee\neg P(a_n) \vee (Q(a_1)\wedge\ldots\wedge Q(a_n))$

$\Leftrightarrow (\neg P(a_1)\vee\ldots\vee\neg P(a_n)\vee Q(a_1))\wedge\ldots\wedge(\neg P(a_1)\vee\ldots\vee\neg P(a_n)\vee Q(a_n))$

$\Leftrightarrow \exists y[\neg P(y)\vee Q(a_1)]\wedge\ldots\wedge\exists y[\neg P(y)\vee Q(a_n)]$

$\Leftrightarrow \forall x\,\exists y[\neg P(y)\vee Q(x)] \Leftrightarrow \forall x\,\exists y\,[P(y)\to Q(x)]$

It is enough that $P(y)$ is false for one element to satisfy this expression, but if it is true for all elements then $Q(x)$ must also be true for all elements.

2. $A\subseteq B \Leftrightarrow A\cup B = B$

if $A\cup B = B$ then

$(\sim A\cup \underbrace{A\cup B}_{=B})\cap(\underbrace{A\cup B}_{=B}\cup\underbrace{(\sim A\cap\sim B)}_{\sim A\setminus B}) =$

$= (\underbrace{\sim A\cup A}_{\Omega}\cup B)\cap(\underbrace{A\cup\sim A}_{\Omega}\cup B)\cap(A\cup \underbrace{B\cup\sim B}_{\Omega}) =$

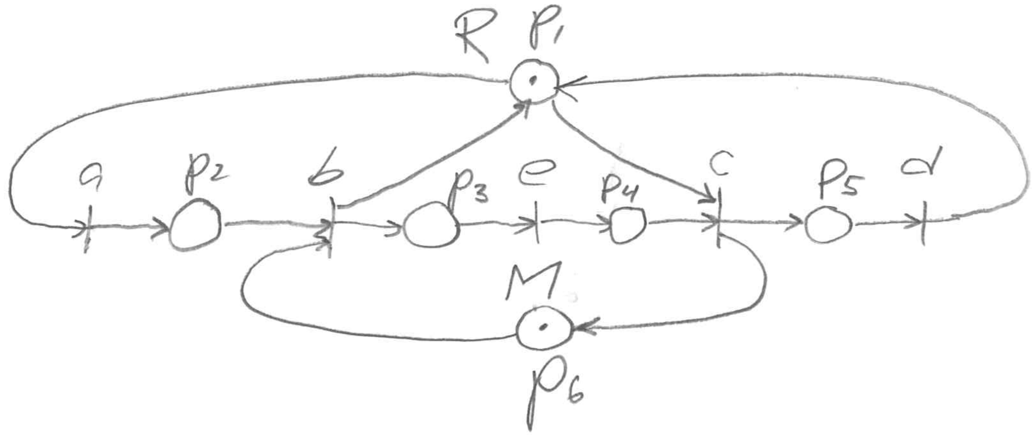
$= (\Omega\cup B)\cap(\Omega\cup B)\cap(A\cup\Omega) =$

$= \Omega\cap\Omega\cap\Omega = \Omega$

3 a) $\mathcal{L}(R) = \overline{(ab + cd)^*(\varepsilon + a + c)}$

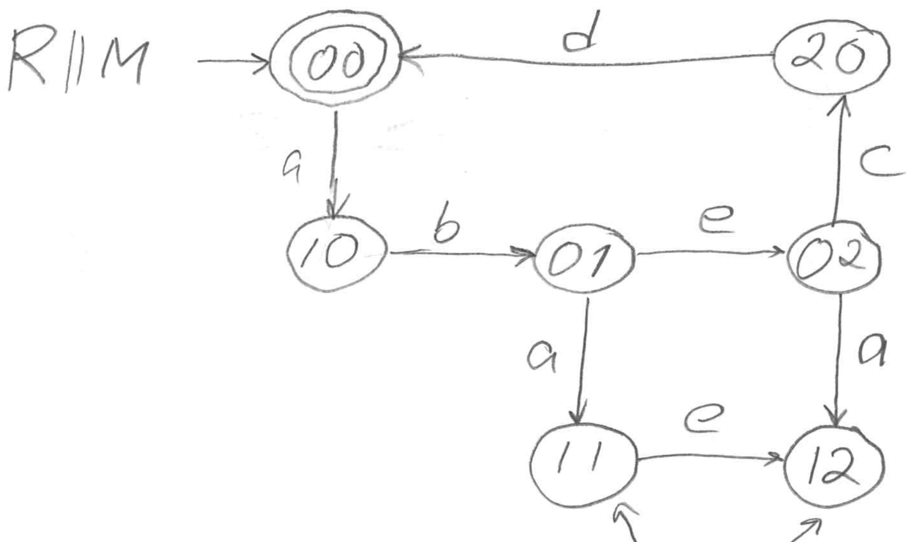
$\mathcal{L}_m(R) = (ab+cd)^*$

b)



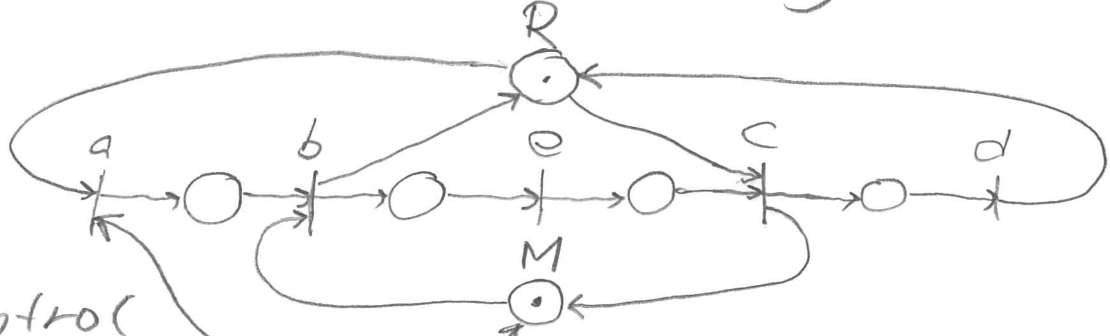
Init $m = [1\ 0\ 0\ 0\ 0\ 1]$ = marked vector

c)



blocking states

d)



control arc that avoids the blocking states.

Executing a is only allowed when the machine M is free (state 0)

4. Evaluate $\varphi_1 = \exists\Box(q \vee r)$ and

$\varphi_2 = \exists\Diamond p$ Then $\varphi = \varphi_1 \wedge \varphi_2$

$[\![\exists\Box(q\vee r)]\!] = \nu Y.\ [\![q\vee r]\!] \cap Pre^{\exists}(Y)$

Maximal fixpoint $\Rightarrow$ $Y_0 = [0,6]$

$Y_1 = [\![q\vee r]\!] \cap Pre^{\exists}([0,6]) =$

$= [0,6]\setminus\{4\} \cap \underbrace{\underbrace{(Pre^{\exists}(\{1\}) \cup Pre^{\exists}(2) \cup Pre^{\exists}(3) \ldots \cup Pre^{\exists}(6))}_{\{0,1\}\cup\{3\}\cup\{0,2,4,6\}\cup\ldots\cup\{5\}}}_{[0,6]}$

$= [0,6]\setminus\{4\}$ $\quad Y_2 = Y_1$

* Generally this is not guaranteed by this procedure, but OK for this simple example

$[\![\exists\Diamond p]\!] = \mu Z.\ [\![p]\!] \cup Pre^{\exists}(Z)$

Minimal fixpoint $\Rightarrow$ $Z_0 = \emptyset$

$Z_1 = [\![p]\!] \cup \emptyset = \{2,4,5\}$

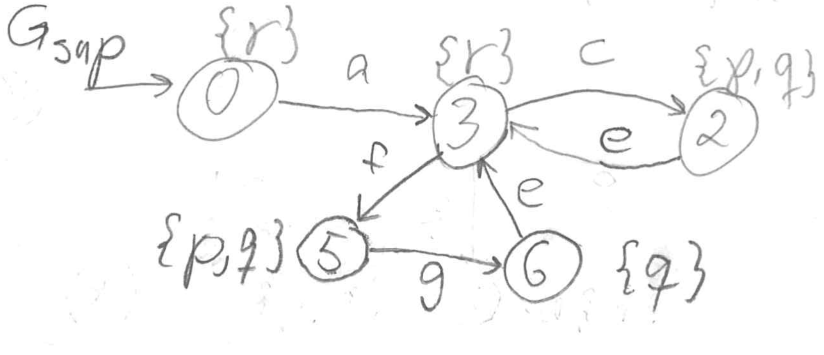
$Z_2 = \{2,4,5\} \cup Pre^{\exists}(\{2,4,5\}) = \{2,4,5\} \cup \{3\} = [2,5]$

$Z_3 = \{2,4,5\} \cup Pre^{\exists}([2,5]) = \{2,4,5\} \cup \{0,2,3,4,6\} = [0,6]\setminus\{1\}$

$Z_4 = \{2,4,5\} \cup Pre^{\exists}([0,6]\setminus\{1\}) = \{2,4,5\} \cup [0,6]\setminus\{1\} = Z_3$

$[\![\varphi]\!] = [\![\varphi_1]\!] \cap [\![\varphi_2]\!] = [0,6]\setminus\{1,4\} = \{0,2,3,5,6\}$

Note that also G_{sup} satisfies both φ_1 and φ_2 (*)



5. $$J^*(x) = \max_{a \in \Sigma(x)} [\underbrace{\rho(x,a) + \gamma J^*(\delta(x,a))}_{Q(x,a)}]$$

$$J^*(2) = \rho_d + \gamma J^*(4) \qquad J^*(3) = \rho_e + \gamma J^*(4)$$

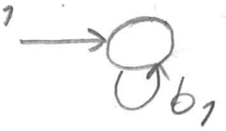
$$\mu(1) = \arg\max_{a \in \{b,c\}} \{ \underbrace{\rho_b + \gamma J^*(2)}_{b}, \underbrace{\rho_c + \gamma J^*(3)}_{c} \}$$

$$= \arg\max_{a \in \{b,c\}} \{ \underbrace{\rho_b + \gamma \rho_d + \gamma^2 J^*(4)}_{b}, \underbrace{\rho_c + \gamma \rho_e + \gamma^2 J^*(4)}_{c} \}$$

Maximum for b when

$$\rho_b + \gamma \rho_d > \rho_c + \gamma \rho_e$$

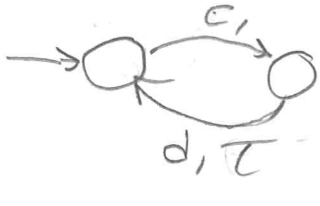
6. $G_1^{\mathcal{A}}$



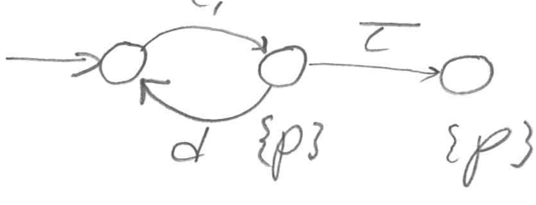
$G_2^{\mathcal{A}}$

$G_{12}^{\mathcal{A}} = (G_1^{\mathcal{A}} \| G_2^{\mathcal{A}})^{\mathcal{A}}$

$G_{13} = G_{12}^{\mathcal{A}} \| G_3^{\mathcal{A}}$



$G_{13} \| G_4$



$$(G_{13} \| G_4)^{\mathcal{A}} = ((G_1^{\mathcal{A}} \| G_2^{\mathcal{A}})^{\mathcal{A}} \| G_3^{\mathcal{A}} \| G_4)^{\mathcal{A}}$$