Discrete Event Systems

Course code: SSY165

Examination 2022-10-22

Time: 8:30-12:30, Location: Johanneberg

Teacher: Bengt Lennartson, phone 0730-79 42 26

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on November 9 and 10, 12:30-13:00 at the division.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Electrical Engineering
Division of Systems and Control
Chalmers University of Technology

1

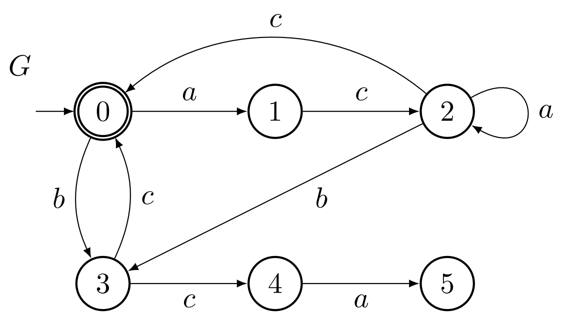
Prove by propositional and temporal logic equivalences that the following temporal logic expression is a tautology

$$\Box p \to (\Box q \to \Box(p \wedge q)) \wedge (\bigcirc(p \to q) \to (\bigcirc p \to \bigcirc q))$$

Some temporal logic equivalences are added in the formula sheet at the end of the exam. (3 p)

2

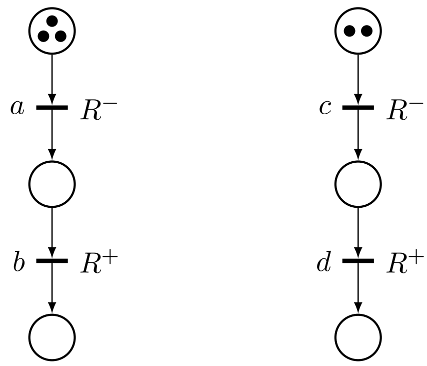
Formulate the language and marked language for the nondeterministic automaton G.



(3 p)

3

Consider the following Petri nets with the shared variable R. The notion $R^{\pm}$ at a transition means that the updated value of R after such a transition is $R' = R \pm 1$. The transition is however only admissible if the next value is 0 or 1. The initial value is $R = 1$.



Formulate individual automata G_i $i = 1, \ldots, 6$, one for each place, and one automaton G_R for the variable R, such that the synchronized system $G_1 \parallel G_2 \parallel \cdots \parallel G_6 \parallel G_R$ models the behavior of the Petri nets above with the shared variable R. (4 p)

4

a) Generate a controllable and nonblocking supervisor for the plant G below with the uncontrollable event set $\Sigma_u = \{b, c, d\}$ and the added specification $Q_m = \{4\}$. Show the resulting automaton after each Coreachability computation. (3 p)

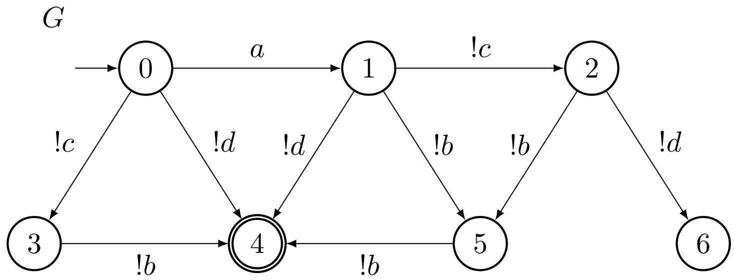
b) Verify that the least fixed point

$$\mu Y.[\![\varphi_m]\!] \cup \text{Pre}^{\exists\forall_u}(Y)$$

where $[\![\varphi_m]\!]$ is the set of marked states and the predecessor operator

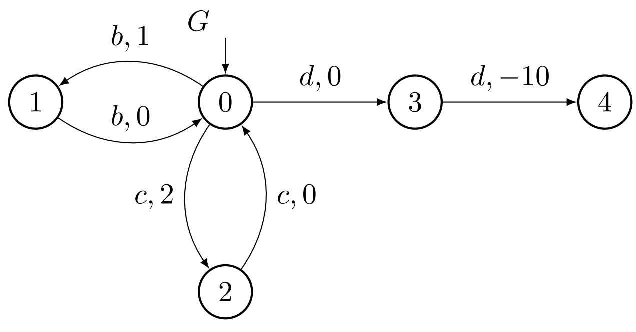
$$\text{Pre}^{\exists\forall_u}(Y) = \{x \,\big|\, \exists a \in \Sigma(x), \forall a \in \Sigma_u(x) : \delta(x,a) \subseteq Y\}$$

generates the same set of supervisor states as the ordinary supervisor synthesis in task 4a). (2 p)



5

Consider the automaton G, including immediate rewards on each transition.



a) Iterate the Q-learning algorithm

$$\widehat{Q}_{k+1}(x,a) = (1-\alpha_k)\widehat{Q}_k(x,a) + \alpha_k\big(r' + \gamma \max_{b \in \Sigma(x')} \widehat{Q}_k(x',b)\big)$$

the first 12 steps for $\alpha_k = 1$ and $\gamma = 0.5$. The action with the largest estimated $\widehat{Q}$-value is chosen (Greedy action) in state 0, except for actions with $\widehat{Q} = 0$ that have higher priority. This strategy is chosen to improve the initial exploration. (3 p)

b) Determine the value the Q-function will converge to for the different actions in state $x = 0$, and compare with the result after 12 steps in task 5 a). Which is the optimal action in the initial state? (2 p)

6

To reduce the number of states, local events that are not involved in any other subsystems can be replaced by the hidden event τ. Any τ transition can then be removed when no alternative transitions are involved in the source state of such τ transitions, and the source and target states have the same state label. Removing a τ transition means that the source and target states are merged into one state.

a) Apply this reduction principle on the following synchronized system

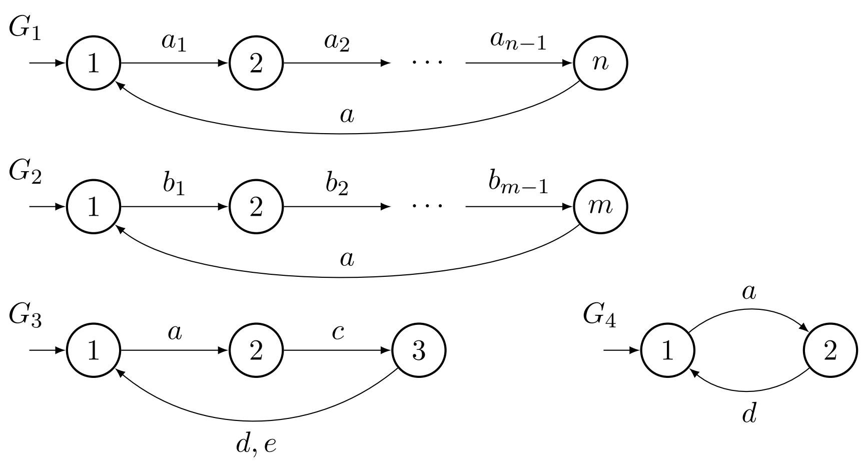
$$G_1 \,\|\, G_2 \,\|\, G_3 \,\|\, G_4,$$

where the individual transition systems are given below, and local τ transitions are identified and removed before every synchronization. Show that the final reduced system

$$((G_1^{\mathcal{A}} \,\|\, G_2^{\mathcal{A}})^{\mathcal{A}} \,\|\, G_3^{\mathcal{A}} \,\|\, G_4)^{\mathcal{A}}$$

only includes three states where one state is a terminal state. The reduction (abstraction operator) $\mathcal{A}$ on any automaton includes the replacement of local events with τ, followed by the state reduction. (4 p)

b) How many states are there in the total system $G_1 \,\|\, G_2 \,\|\, G_3 \,\|\, G_4$ without abstraction? (1 p)



Solution Exam DES 2022-10-22

1. $\Box p \to (\Box q \to \Box(p \wedge q)) \wedge (\bigcirc(p \to q) \to (\bigcirc p \to \bigcirc q))$

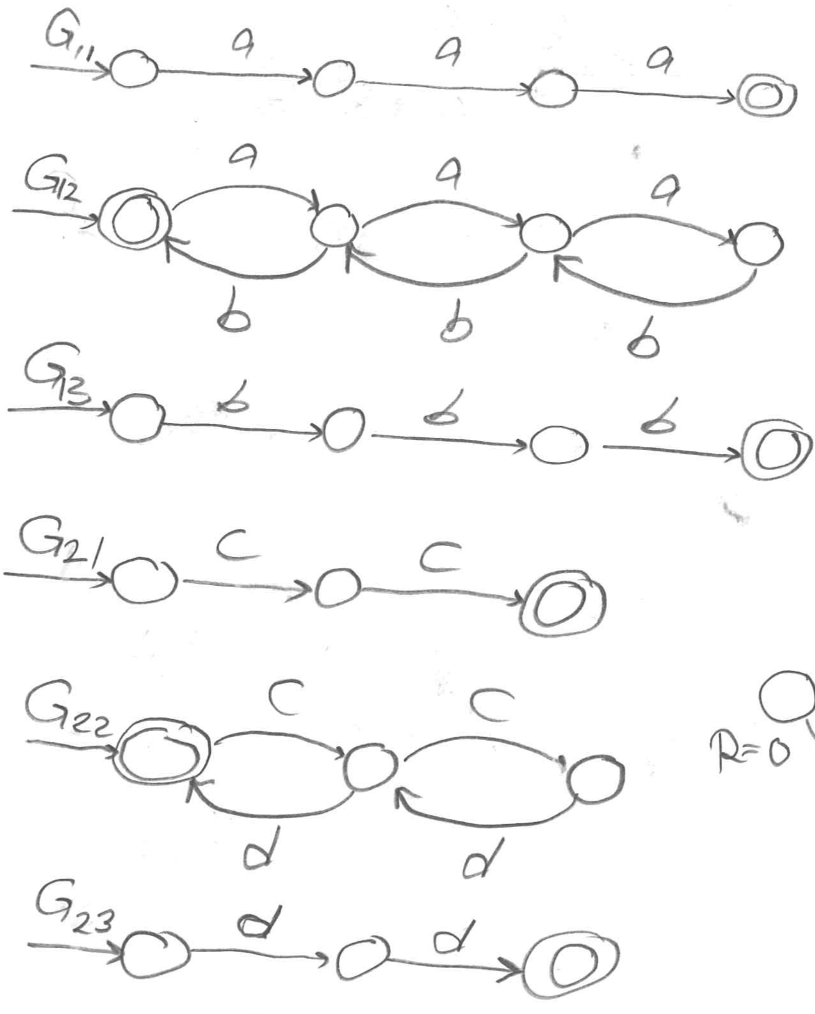
$$\equiv \neg\Box p \vee \underbrace{(\neg\Box q \vee (\Box p \wedge \Box q))}_{(\neg\Box q \vee \Box p)\wedge\underbrace{(\neg\Box q \vee \Box q)}_{T}} \wedge \underbrace{(\neg\bigcirc(p\to q) \vee \bigcirc(p \to q))}_{T}$$

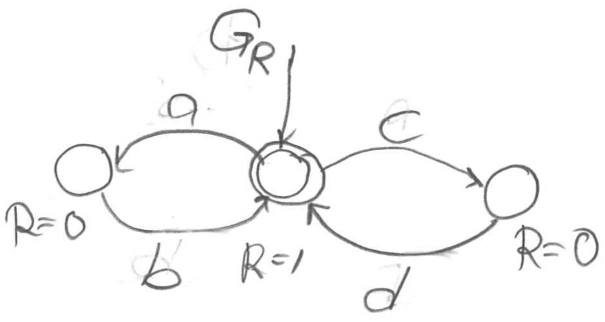
$$\equiv (\neg\Box p \vee \Box p \vee \neg\Box q) \wedge T \equiv T \vee \neg\Box q \equiv T$$

2. $\mathcal{L}(G) = \overline{(aca^*(\varepsilon+b)c+bc)^*(aca^*+\varepsilon)bca}$

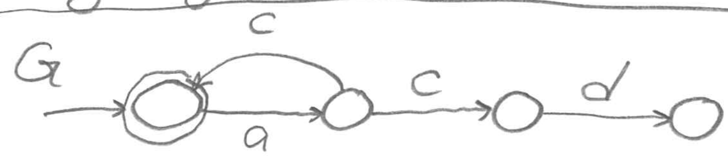
$\mathcal{L}_m(G) = (aca^*(\varepsilon+b)c+bc)^*$

3. See also the next extra page with comments on the nondeterminism.





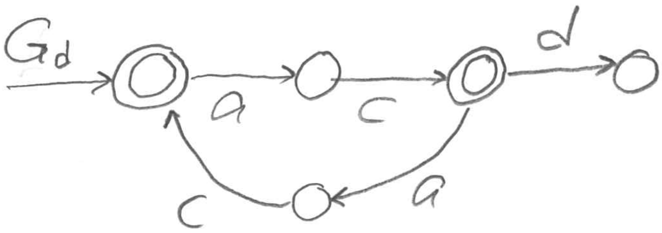
Language and nondeterminism



$$L(G) = \overline{(ac)^* ac\, d} = \overline{(ac)^+ d}$$

$$L_m(G) = (ac)^*$$

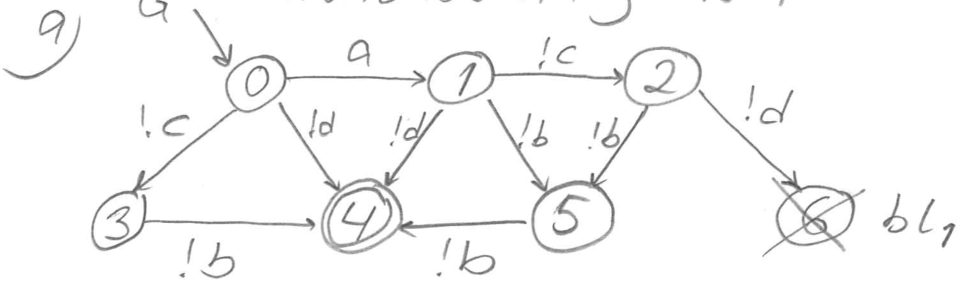
A deterministic automaton G_d with the same language



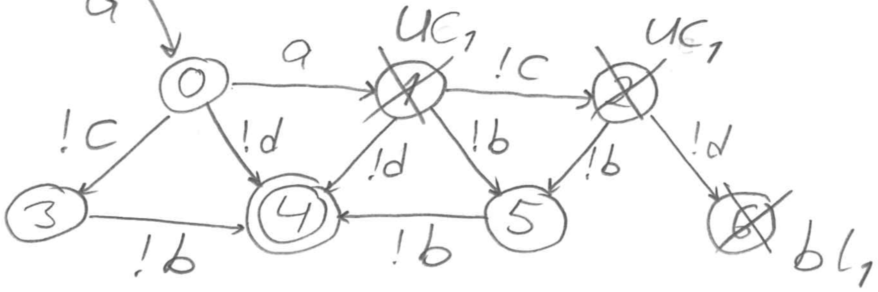
Thus, a language can not explicitly model desired nondeterminism, since a regular language with the three operators $+, \cdot, *$ can always be translated to a deterministic automaton.

4.

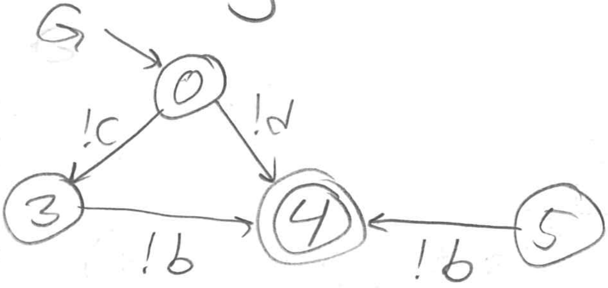
a) Nonblocking $k=1$



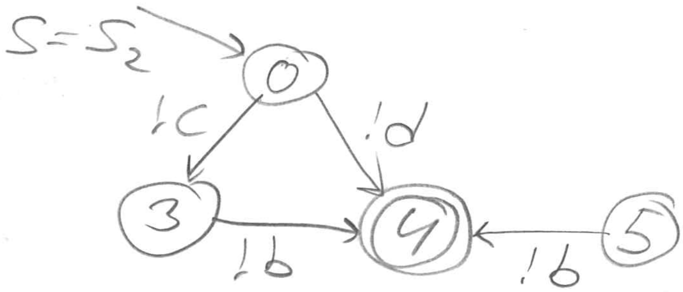
controllable $k=1$



Nonblocking $k=2$



Controllable $k=2$



b)

$$\mu Y.\, \Psi(Y) \qquad \Psi(Y) = [\![\varphi_m]\!] \cup Pre^{\exists\forall u}(Y)$$

$Y_0 = \emptyset \qquad Y_1 = \{4\} \cup Pre^{\exists\forall u}(\emptyset) = \{4\}$

$Y_2 = \{4\} \cup Pre^{\exists\forall u}(\{4\}) = \{4\} \cup \{3,5\} = \{3,4,5\}$

not $x=0$ since $3 \notin Y_1$

not $x=1$ since $5 \notin Y_1$

$Y_3 = \{4\} \cup Pre^{\exists\forall u}\{3,4,5\} = \{4\} \cup \{0,3,5\} = \{0,3,4,5\}$

not $x=1$ since $2 \notin Y_2$

not $x=2$ since $6 \notin Y_2$

$x = 1,2$ not included, see Y_3

$Y_4 = \{4\} \cup Pre^{\exists\forall u}\{0,3,4,5\} = \{4\} \cup \{0,3,5\} = \{0,3,4,5\} = Y_3 = Y_\omega$

5. a) $\hat{Q}_{k+1}(x,a) = r' + \gamma \max_{b \in \Sigma(x')} \hat{Q}_k(x',b)$

k	x	a	x'	r'	$\hat{Q}_k(x',b)$	$\hat{Q}_{k+1}(x,a)$
1	0	b	1	1	$\hat{Q}(1,b)=0$	$\hat{Q}(0,b)=1$
2	1	b	0	0	$\hat{Q}(0,b)=1, \hat{Q}(0,c)=\hat{Q}(0,d)=0$	$\hat{Q}(1,b)=\gamma$
3	0	c	2	2	$\hat{Q}(2,c)=0$	$\hat{Q}(0,c)=2$
4	2	c	0	0	$\hat{Q}(0,b)=1, \hat{Q}(0,c)=2, \hat{Q}(0,d)=0$	$\hat{Q}(2,c)=2\gamma$
5	0	d	3	0	$\hat{Q}(3,d)=0$	$\hat{Q}(0,d)=0$
6	3	d	4	-10	$x'=4$, terminal state	$\hat{Q}(3,d)=-10$
7	0	d	3	0	$\hat{Q}(3,d)=-10$	$\hat{Q}(0,d)=-10\gamma$
8	3	d	4	-10	$x'=4$, terminal state	$\hat{Q}(3,d)=-10$
9	0	c	2	2	$\hat{Q}(2,c)=2\gamma$	$\hat{Q}(0,c)=2(1+\gamma^2)$
10	2	c	0	0	$\hat{Q}(0,b)=1, \hat{Q}(0,c)=2(1+\gamma^2), \hat{Q}(0,d)=-10\gamma$	$\hat{Q}(2,c)=2(\gamma+\gamma^3)$
11	0	c	2	2	$\hat{Q}(2,c)=2(\gamma+\gamma^3)$	$\hat{Q}(0,c)=2(1+\gamma^2+\gamma^4)$
12	2	c	0	0	$\hat{Q}(0,b)=1, \hat{Q}(0,c)=2(1+\gamma^2+\gamma^4), \hat{Q}(0,d)=-10\gamma$	$\hat{Q}(2,c)=2(\gamma+\gamma^3+\gamma^5)$

b) $J^*(0) = \max\{ \underbrace{1+\gamma J^*(1)}_{\text{b-action}}, \underbrace{2+\gamma J^*(2)}_{\text{c-action}}, \underbrace{0+\gamma J^*(3)}_{\text{d-action}} \}$

$J^*(1) = \gamma J^*(0) = J^*(2) \qquad J^*(3) = -10 + \gamma J^*(4) = -10$

$\therefore J^*(0) = \max\{ \overbrace{1+\gamma^2 J^*(0)}^{Q(0,b)}, \overbrace{2+\gamma^2 J^*(0)}^{Q(0,c)}, \overbrace{-10\gamma}^{Q(0,d)} \} = 2+\gamma^2 J^*(0)$

$\Rightarrow (1-\gamma^2) J^*(0) = 2$, i.e. $J^*(0) = \frac{2}{1-\gamma^2}$

$Q_\infty(0,b) = 1 + \gamma^2 J^*(0) = 1 + \frac{2\gamma^2}{1-\gamma^2} = \frac{1+\gamma^2}{1-\gamma^2}$

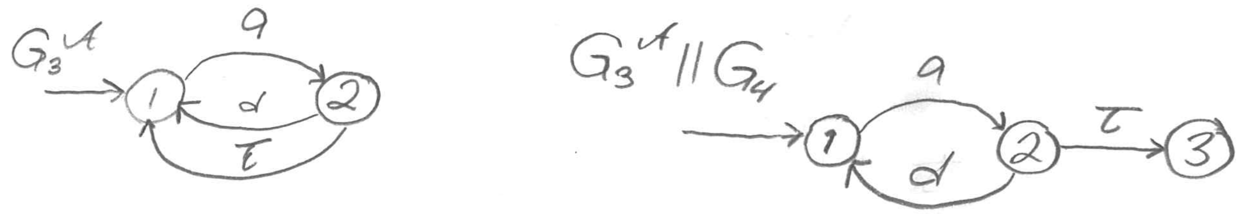
$Q_\infty(0,c) = 2 + \gamma^2 J^*(0) = 2 + \frac{2\gamma^2}{1-\gamma^2} = \frac{2}{1-\gamma^2}$

$Q_\infty(0,d) = -10\gamma$

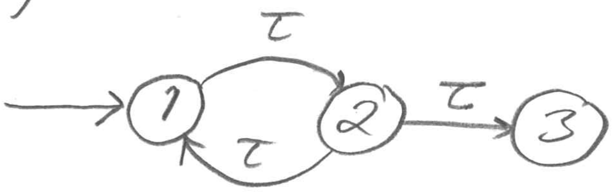
Optimal action in state 0 is $\max_{a \in \Sigma(0)} Q_\infty(0,a) = c$

6. a) In G_1 and G_2 the event a is shared, while the rest are local $\Rightarrow$

In G_3 the events c and e are local $\Rightarrow$

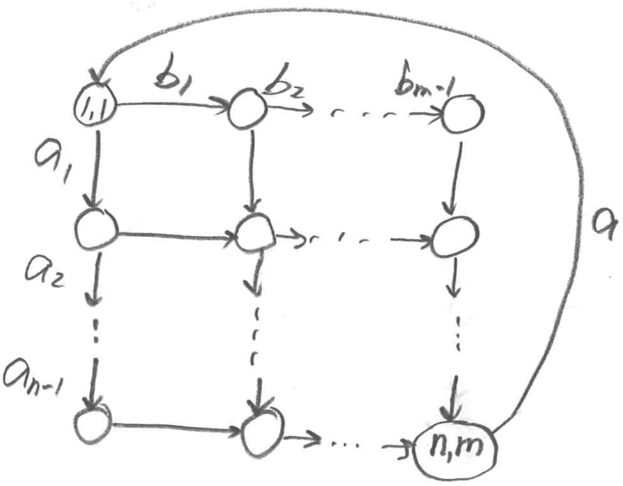


$$((G_1^{\mathcal{A}} \| G_2^{\mathcal{A}})^{\mathcal{A}} \| G_3^{\mathcal{A}} \| G_4)^{\mathcal{A}}$$



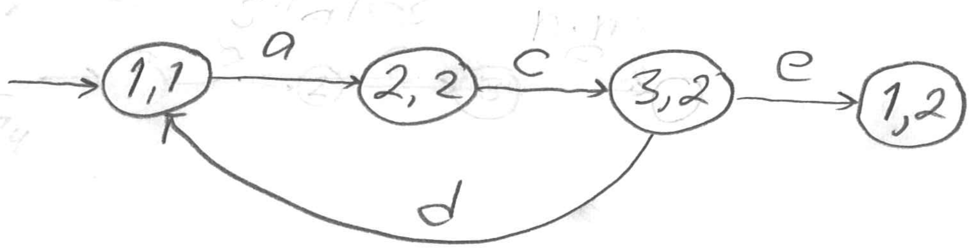
b) $G_1 \| G_2$

state space of $G_1 \| G_2$ is denoted X_{12} ($|X_{12}| = nm$)

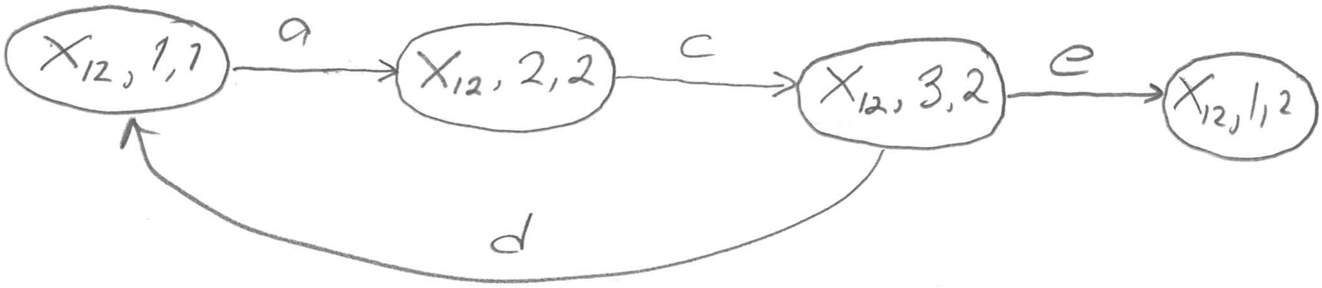


Number of states $= n \cdot m$

$G_3 \| G_4$



$G_1 \| G_2 \| G_3 \| G_4$



Total number of states =

$4 \cdot |X_{12}| = 4nm$

$n = m = 2$