Logic, Learning, and Decision

Course code: SSY165

Examination 2024-10-29

Time: 8:30-12:30, Location: Johanneberg

Teacher: Bengt Lennartson, phone 0730-79 42 26

The examination includes 25 points, where grade three requires 10 points, grade four 15 points and grade five 20 points.

The result of this examination is announced and inspection of the grading is done on November 14 and 15, 12:30-13:00 at the division.

Allowed aids at the examination:

- Standard mathematical tables such as Beta, see also formulas in the end of this examination.
- Pocket calculator.

Good luck!

Department of Electrical Engineering
Division of Systems and Control
Chalmers University of Technology

1

Prove that

$$\forall x[p(x)] \rightarrow q \Leftrightarrow \exists x[p(x) \rightarrow q]$$

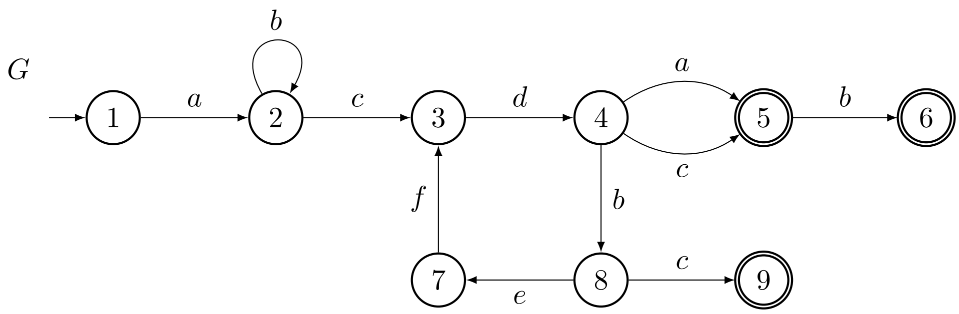
by assuming a universal set Ω for the variable x, including a finite number of arbitrary elements

$$\Omega = \{a_1, a_2, \cdots, a_n\}$$

(2 p)

2

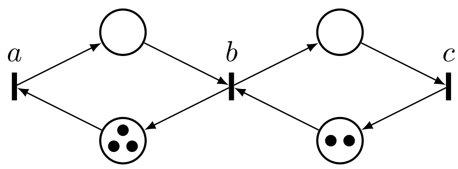
Formulate the language $\mathcal{L}(G)$ and the marked language $\mathcal{L}_m(G)$ for the following automaton.



(3 p)

3

The following Petri net models two buffers, the first one with capacity 3 and the second one with capacity 2.



a) Generate the reachability graph for this Petri net. (2.5 p)

b) How many states are there in this system? (0.5 p)

c) How many transitions in the reachability graph have label a, b, and c? (0.5 p)

d) How many tokens are there in each state, and how does this number relate to the total capacity of the buffer system? (0.5 p)

e) When the capacity of the two buffers are m_1 and m_2, respectively, what is then the sum of tokens in each state of the corresponding reachability graph? (1 p)

4

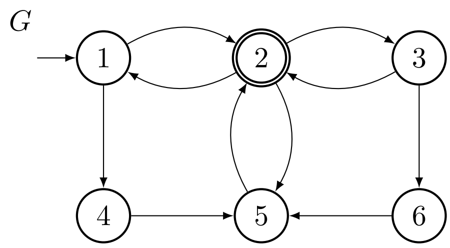
a) Formulate a plant automaton P and a specification automaton Sp, such that $P||Sp$ is blocking and uncontrollable with respect to the plant P. No restrictions on the number of states for Sp, while P must include at least five states. (2 p)

b) Verify that $P||Sp$ is blocking and uncontrollable with respect to the plant P, for your specific choice of P and Sp. (1 p)

c) Generate a controllable and nonblocking supervisor S, for the chosen P and Sp, by the fixed point algorithm presented in the lecture notes. Show the resulting automaton after each Coreachability computation. (2 p)

5

Consider the following automaton, where the state label of the marked state satisfies the atomic proposition p.

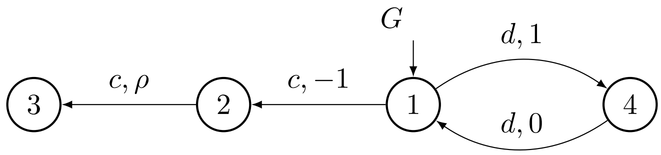


a) Motivate why this automaton satisfies the LTL formula $\Box\Diamond p$ but not $\bigcirc p$. (2 p)

b) Show by μ-calculus that the CTL formula $\forall\Box\exists\Diamond p$ holds for this automaton. (3 p)

6

Consider the following discrete state-space model G, including different rewards for the various state transitions.



a) Iterate the Q-learning algorithm

$$\widehat{Q}_{k+1}(x,a) = (1-\alpha_k)\widehat{Q}_k(x,a) + \alpha_k\big(r' + \gamma \max_{b\in\Sigma(x')} \widehat{Q}_k(x',b)\big)$$

the first 8 steps for $\alpha_k = 1$, $\gamma = 0.5$ and $\rho = 4$. The action with the largest estimated $\widehat{Q}$-value is chosen (Greedy action) in state 1, except for actions with the initial value $\widehat{Q} = 0$ that have higher priority. This strategy is chosen to improve the initial exploration.

(2 p)

b) Determine the value the Q-function will converge to in state 1, and compare with the result after 8 steps. Which is the optimal action in the initial state, and which immediate reward ρ is required to make action c the optimal decision in state 1?

(2 p)

c) Determine the interval of ρ for which the estimated Q-function after eight steps generates a wrong decision. (1 p)

Solution Logic, Learning & Decision 2024-10-29 BL

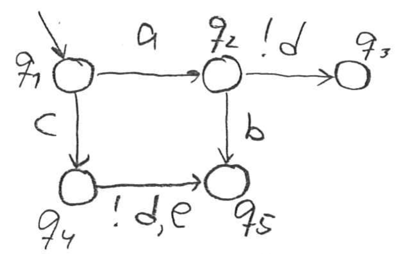
1. $\forall x[p(x)] \to q \Leftrightarrow (p(a_1) \wedge \dots \wedge p(a_n)) \to q \Leftrightarrow$
$\neg p(a_1) \vee \dots \vee \neg p(a_n) \vee q \Leftrightarrow \neg p(a_1) \vee q \vee \dots \vee \neg p(a_n) \vee q \Leftrightarrow$
$(p(a_1) \to q) \vee \dots \vee (p(a_n) \to q) \Leftrightarrow \exists x[p(x) \to q]$

2. $L_m(G) = ab^*c(dbef)^*d((a+c)(\varepsilon+b)+bc)$

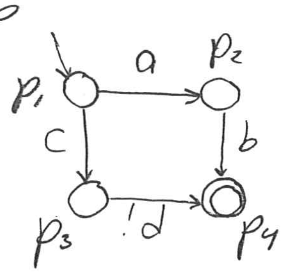
$L(G) = \overline{L_m(G)}$

3. See next page

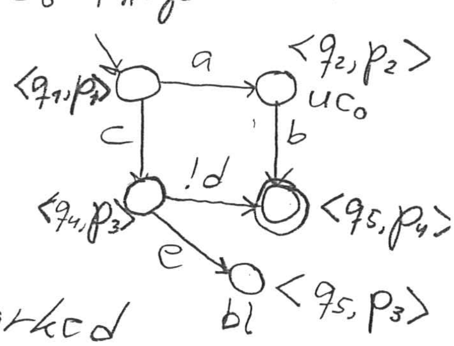
4. a) P



Sp



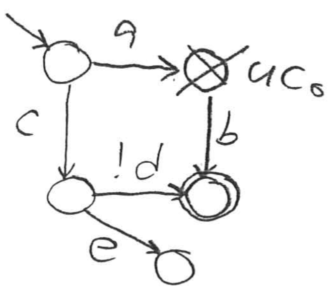
$S_0 = P \| Sp$

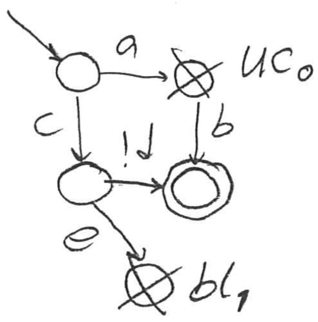


b) $\langle q_5, p_3 \rangle$ is blocking since the marked state $\langle q_5, p_4 \rangle$ cannot be reached from this state. $\langle q_2, p_2 \rangle$ is uncontrollable since there is an uncontrollable transition $q_2 \xrightarrow{!d} q_3$ in P but no outgoing transition from p_2 with the uncontrollable event d.

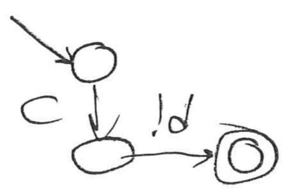
c) S_0 inc uc (uncontrollable) state

bl_i = blocking state iteration i

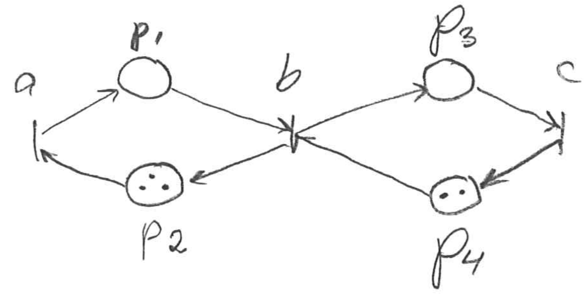




$S_2 = S_1 = S$

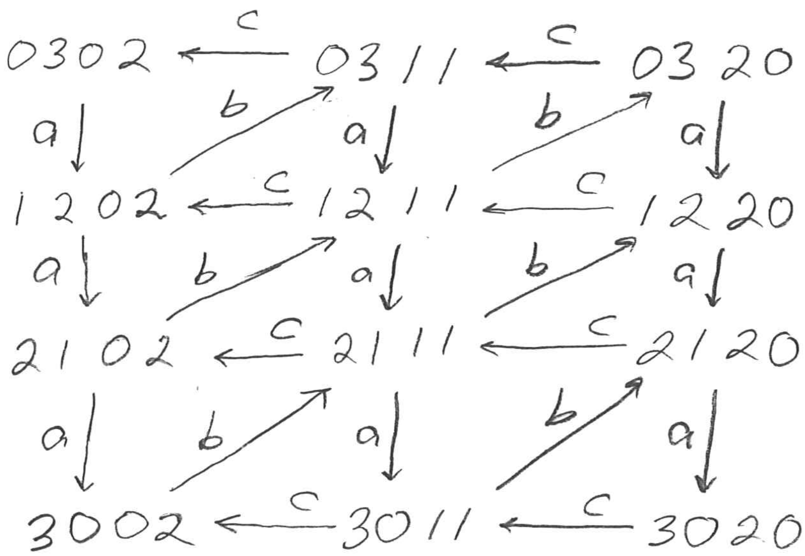


3.



$m_0 = [0\ 3\ 0\ 2]^T$

a)

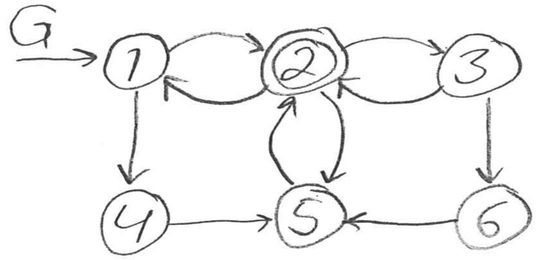


b) 12 states

c) 9 a-labels, 6 b-labels, 8 c-labels

d) 5 tokens which corresponds to the total capacity of the system = capacity Buffer 1 + capacity Buffer 2 = 3 + 2 = 5

e) Sum of tokens in each state is $m_1 + m_2$.

5. 

$\lambda(2) = \{p\}$

a) From the initial state 1, all paths will infinitely often pass state 2 and $\lambda(2) = \{p\} \Rightarrow G \models \Box\Diamond p$

From the initial state, the next state is 2 where $\lambda(2) = \{p\}$, but also 4 and $\lambda(4) = \emptyset$. Thus, $G \not\models \bigcirc p$

b) $[\![\exists\Diamond p]\!] = \mu Y.\Psi(Y)$, $\quad \Psi(Y) = [\![p]\!] \cup Pre^{\exists}(Y)$, $Y_0 = \emptyset$

$Y_1 = \{2\} \cup Pre^{\exists}(\emptyset) = \{2\}$

$Y_2 = \{2\} \cup Pre^{\exists}(\{2\}) = \{2\} \cup \{1,3,5\} = \{1,2,3,5\}$

$Y_3 = \{2\} \cup Pre^{\exists}(\{1,2,3,5\}) = \{2\} \cup \{2,1,3,5,4,6\} = [1,6] = \mathbb{X}$

$Y_4 = \{2\} \cup Pre^{\exists}(\mathbb{X}) = \mathbb{X} = Y_3 = Y^{\omega} = [\![\underbrace{\exists\Diamond p}_{w}]\!]$

$[\![\forall\Box\exists\Diamond p]\!] = [\![\forall\Box w]\!]$ where $[\![w]\!] = Y^{\omega} = \mathbb{X}$

$[\![\forall\Box w]\!] = \nu Z.\Psi(Z)$, $\quad \Psi(Z) = [\![w]\!] \cap Pre^{\forall}(Z)$, $Z_0 = \mathbb{X}$

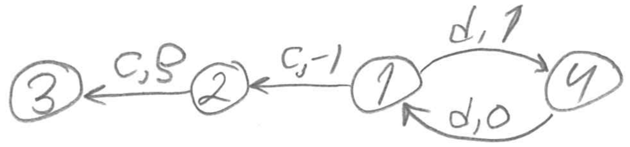
$Z_1 = \mathbb{X} \cap Pre^{\forall}(\mathbb{X}) = \mathbb{X} \cap \{2,1,3,5,4,6\} = \mathbb{X}$

$Z_2 = \mathbb{X} \cap Pre^{\forall}(\mathbb{X}) = \mathbb{X} = Z_1 = Z^{\omega} =$

$= [\![\forall\Box w]\!] = [\![\forall\Box\exists\Diamond y]\!]$

$\therefore$ The initial state of G satisfies $\forall\Box\exists\Diamond p \Rightarrow G \models \forall\Box\exists\Diamond p$

6.



a) $\hat{Q}_{k+1}(x,a) = r' + \gamma \max_b \hat{Q}(x',b)$ $\quad \rho = 4$, $\gamma = 0.5$

Choose c and d two times each in state x = 1

k	x	a	r'	x'	$\hat{Q}(x',b)$	$\hat{Q}(x,a)$
1	1	c	-1	2	$\hat{Q}(2,c)=0$	$\hat{Q}(1,c)=-1$
2	2	c	4	3	$\hat{Q}(3,\cdot)=0$	$\hat{Q}(2,c)=4$
3	1	c	-1	2	$\hat{Q}(2,c)=4$	$\hat{Q}(1,c)=-1+4\gamma=1$
4	2	c	4	3	$\hat{Q}(3,\cdot)=0$	$\hat{Q}(2,c)=4$
5	1	d	1	4	$\hat{Q}(4,d)=0$	$\hat{Q}(1,d)=1$
6	4	d	0	0	$\hat{Q}(1,d)=1$, $\hat{Q}(1,c)=1$	$\hat{Q}(4,d)=\gamma$
7	1	d	1	4	$\hat{Q}(4,d)=\gamma$	$\hat{Q}(1,d)=1+\gamma^2$
8	4	d	0	0	$\hat{Q}(1,d)=1+\gamma^2$; $\hat{Q}(1,c)=1$	$\hat{Q}(4,d)=\gamma+\gamma^3$

b) $Q(x,a) = r' + \gamma Q_{max}(x')$ $\quad \gamma = 0.5,\ \rho = 4$

$Q(2,c) = \rho \quad Q(1,c) = -1 + \gamma\rho = -1 + 0.5 \cdot 4 = 1$

When $Q(1,d) > Q(1,c) \Rightarrow Q(4,d) = \gamma Q(1,d)$

$Q(1,d) = 1 + \gamma Q(4,d) = 1 + \gamma^2 Q(4,d)$

$Q(1,d) = \frac{1}{1-\gamma^2} = \frac{1}{0.75} = 1.33 > Q(1,c) = 1$

$\because \rho = 4 \Rightarrow$ optimal decision for $x=1$ is $\mu(x) = d$ for $\rho = 4$.

Since $\hat{Q}(1,d) = 1+\gamma^2 = 1.25 > \hat{Q}(1,c) = 1$ also the estimate $\hat{Q}$ gives the correct decision.

c) $Q(1,c) = -1 + 0.5\rho > Q(1,d) = \frac{1}{1-\gamma^2} = 1.33$

for $\rho > 2.33/0.5 = 4.66$.

$\because \rho > 4.66 \Rightarrow \mu(1) = c$

c) $\hat{Q}(1,c) = -1 < \hat{Q}(1,d) \Rightarrow$ estimated decision after 8 steps is $\hat{\mu}(1) = d$ for all ρ while optimal decion is $\mu(1) = c$ for $\rho > 4.66$. Thus, the estimated decision is wrong for $\rho > 4.66$